

# An Intermediation-Based Model of Exchange Rates \*

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## Abstract

We develop a general equilibrium model of decentralized international financial markets. In our model, financial intermediaries bargain with their customers and extract endogenous rents for providing access to foreign claims. The behavior of intermediaries, by tilting state prices, generates a non-linear risk structure in exchange rates. We use this risk structure to explicitly derive (i) a link between monetary and stabilization policies and safe haven properties of exchange rates; (ii) the global monetary spillover matrix; and (iii) deviations from covered interest rate parity (CIP), and show how all these effects depend on international intermediation capacities.

**Keywords:** exchange rates, dollar, covered interest parity deviations, currency skew, currency crashes

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The goal of this paper is to develop a macroeconomic general equilibrium model in which international financial markets are subject to intermediation frictions. In our model, intermediaries use their market power to extract rents from their customers for providing them with access to trading foreign financial instruments. This rent extraction distorts international risk sharing and alters the dynamics of international risk premia and exchange rates. We show how this simple intermediation friction helps account for some of the major anomalies in foreign exchange and international capital markets including the safe haven properties of exchange rates, currency crash risk, and the breakdown of covered interest parity (CIP).

International financial markets are highly decentralized. The trading of key financial instruments – such as sovereign and corporate bonds, spot foreign exchange (FX) rates, FX forwards and swaps, and most other derivatives used for hedging purposes typically occurs over-the-counter (OTC) through financial intermediaries.<sup>1</sup> Trading in such markets is subject to frictions, whereby a handful of global intermediaries exert significant market power.<sup>2</sup> To study the effect of these market imperfections on exchange rates and the macroeconomy, we introduce an imperfectly competitive intermediation sector into a classical cash-in-advance model a-la [Lucas \(1982\)](#). Our model features an economy with multiple countries and partially integrated financial markets. Each country is populated by two classes of agents, customers (households) and specialists (intermediaries). We introduce some realistic features of segmentation in our model. While customers have free access to local markets for simple local securities (such as local nominal risk free bonds and the local stock market), they have to

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<sup>1</sup>Indeed, a large part of the trading in global securities and derivatives markets occurs over-the counter, with bank dealers as major suppliers of intermediation services. For example, daily turnover in interest rate swaps reached almost USD 2 trillion per day in April 2016, while daily trading volume in the global FX market exceeds USD 5 trillion, according to the most recent BIS statistics on global OTC derivatives markets. See, [BIS \(2016\)](#). Trading in global OTC markets dwarfs the volume that is traded, e.g. on equities or futures exchanges. In OTC markets, an identical asset is typically traded at different prices at a given point in time, depending on the identity of the trading counter-parties.

<sup>2</sup>See, for example, [Hau et al. \(2017\)](#) who provide evidence for significant rent extraction in the foreign exchange derivatives markets. According to [Hau et al. \(2017\)](#), “A corporate client at the 75th percentile of average transaction costs pays a roughly 12 times larger spread than a corporate client at the 25th percentile.”

go via intermediaries in order to gain access to foreign assets and financial instruments in the dealer-to-customer (D2C) market segment. Intermediaries charge for lending their balance sheet to customers. Upon contact, they take into account customers' optimal demand for foreign financial assets and use their bargaining power to extract rents and charge markups for providing insurance against (or, speculative bets on) different states of the world economy. At the same time, intermediaries use the dealer-to-dealer (D2D) market to rebalance their inventories, manage their balance sheet and share risks.

Since intermediaries are the marginal investors in international financial markets, their wealth dynamics emerge as key determinants of international risk premia and the behavior of exchange rates. Since intermediaries' wealth dynamics are determined by the markups they charge to customers, in equilibrium these markups enter directly into the global pricing kernel and, hence, emerge as an important determinant of exchange rates.<sup>3</sup> When a high markup state is realized, intermediaries' wealth goes up, while their marginal utility of consumption drops. Due to the cash-in-advance constraint, the value of the local currency moves one-to-one with this marginal utility, implying that the currency depreciates and (potentially) crashes in a high markup state. Thus, currency crashes become a self-fulfilling prophecy that arise due to the inability of competitive intermediaries to internalize the pecuniary externality generated by their markups. When such crashes materialize at times of low ex-ante intermediation capacity, their severity is amplified even further.<sup>4</sup>

In our model, heterogeneity in risk properties of exchange rates is to a large extent determined by two factors: differences in the conduct of monetary and stabilization policies across countries, as well as differences in intermediation capacities. In a way, intermediaries and the monetary authority play complementary roles by determining the allocation of nominal risk across states. The monetary authority does so by pursuing stabilization policies

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<sup>3</sup>Specifically, local pricing kernels are given by local intermediaries' marginal utilities, while exchange rates are given by the ratio of these kernels.

<sup>4</sup>In particular, consumption risk in our model is also endogenous and depends on the level of financial development (captured by the intermediation capacity), as in [Acemoglu and Zilibotti \(1997\)](#).

that adjust the monetary policy stance in response to local and global shocks. By changing the risk structure of the economy, such policies affect customers' demand for insurance from intermediaries. In turn, intermediaries determine the price of this insurance which feeds back into consumption allocation and determines the passthrough of the stabilization policy into state prices. This changes the nature of the risk allocation between customers and intermediaries, and impacts intermediaries' demand for currencies, determining the transmission of stabilization policies into the exchange rates. One of our major goals is to understand how these channels influence the response of exchange rates to global macroeconomic conditions; in particular, the so-called safe haven properties of exchange rates.

The term “safe haven” is commonly used for currencies that tend to appreciate at times of global economic downturns, usually accompanied with stock market crashes. In standard, frictionless monetary models, policies that aggressively ease monetary policy in global economic downturns naturally lead to currency depreciation: An increase in the money supply leads to an immediate drop in the value of money. These effects are particularly pronounced for countries with large intermediation sectors because these countries naturally serve as insurance providers to the rest of the world and hence suffer the most during crisis periods. This is what [Maggiori \(2013\)](#) calls the “reserve currency paradox”. We show that intermediation markups have the potential to (at least partially) resolve this paradox: While countries with larger intermediation sector indeed suffer more from global downturns, their markups are also more sensitive to global conditions; when this markup channel is strong enough, it overturns the standard risk sharing channel and implies that the currency appreciates.

Our model also allows us to study the impact of non-fundamental shocks in the form of monetary policy uncertainty on exchange rates. Customers in a country where monetary policy is highly uncertain contact intermediaries to buy insurance against this source of

uncertainty. Intermediaries charge markups for providing this insurance, affecting customers' ability to allocate risk across states and forcing the currency to depreciate at time of a global crash. Thus, the currency of a country where there is little monetary policy uncertainty naturally emerges as a safe haven. The underlying mechanism is characteristic to our model: customers' expectations about future policy create demand pressure in the D2C market, determining equilibrium markups and the risk properties of exchange rates.

A key feature of our model is heterogeneity in countries' risk profiles which emerges endogenously. We show that customers in a particular country shift between risk-on or risk-off behaviour depending on how they perceive the co-movement of their future wealth with the contribution of their own net worth to global wealth (in trade-weighted terms). In the extreme case, the sign of this co-movement creates a dichotomy of countries: The impact of stabilisation policies on exchange rates and country welfare is opposite for countries with positive and negative expected co-movement.

In our model, customers willing to borrow or lend in a foreign currency cannot do so directly and have to go through intermediaries. For example, they can do it by borrowing in the local currency and then entering an FX swap contract with the intermediary in order to borrow dollars synthetically; the corresponding indirect rate of borrowing dollars may be quite different from the rate at which intermediaries can borrow dollars directly. Such deviations from covered interest rate parity (CIP) have been a pervasive phenomenon in the post-crisis period.<sup>5</sup>

The sign of these deviations is determined by the joint desire of customers' and intermediaries to borrow or lend in US dollars, which (due to market fragmentation) creates a price pressure in the swap market. We use our model to derive the *fundamental markup equation* that links CIP deviations to the intermediation capacity, defined as the ratio of

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<sup>5</sup>CIP states that the interest rate differentials implicit in foreign exchange swap markets coincide with the corresponding differential in money market rates in the two countries. The breakdown of CIP even for some of the world's most liquid currency pairs is one of the most surprising developments in global financial markets over the past few years. See, for example, [Du et al. \(2016\)](#), [Avdjiev et al. \(2016\)](#), [Borio et al. \(2016\)](#), and [Rime et al. \(2017\)](#).

intermediary-to-customer net worth. The fundamental markup equation implies that (1) the size of the basis is inversely proportional to intermediation capacity; and (2) the sign of the basis is opposite to the sign of the covariance of the intermediation capacity with the US dollar. The intuition is as follows. When deciding in which currency to borrow, customers put a lot of weight on the states in which their net worth is low and their marginal utilities are high, making debt repayment costly. When the covariance of the intermediation capacity with the dollar is negative, customers' net worth is low (relative to that of intermediaries) precisely in the states in which US Dollar is weak. Thus, customers find borrowing in dollars attractive. Intermediaries exploit customers' demand for dollar funding and charge a markup (premium) for lending in dollars. A drop in intermediation capacity amplifies the difference in marginal utilities and hence increases the size of the basis. Importantly, in our model we are able to show how these effects arise endogenously and how they are linked to differences in monetary policy conduct across countries.

**Roadmap.** The remainder of the paper is structured as follows. Section 1 provides an overview of the relevant literature. Section 2 describes the model. Section 3 provides the equilibrium characterization.<sup>6</sup> Section 4 investigates the link between intermediation frictions and various exchange rate anomalies. Section 5 concludes.

## 1 Literature Review

The literature on general equilibrium models of exchange rates is vast. Most papers either assume complete international financial markets (see, for example, Lucas (1982); Cole and Obstfeld (1991), Dumas (1992); Backus et al. (1992); Backus and Smith (1993); Obstfeld and Rogoff (1995); Pavlova and Rigobon (2007); Verdelhan (2010); Colacito and Croce (2011); Hassan (2013)) or an exogenously specified market incompleteness in the form of

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<sup>6</sup>Section A.1 solves for the equilibrium in the frictionless case which acts as an important benchmark case and reference throughout the paper.

portfolio constraints (see, for example, [Chari et al. \(2002\)](#); [Corsetti et al. \(2008\)](#); [Pavlova and Rigobon \(2008\)](#)), unspanned risk factors ([Pavlova and Rigobon \(2010, 2012\)](#), [Farhi and Gabaix \(2016\)](#), [Brunnermeier and Sannikov \(2017\)](#)) or limits to market participation ([Alvarez et al. \(2002, 2009\)](#) and [Bacchetta and Van Wincoop \(2010\)](#)). By contrast, in our model market incompleteness and limits to international risk sharing are endogenous, and are determined by equilibrium intermediation markups.

The most closely related to ours are the papers by [Maggiore \(2013\)](#), [Gabaix and Maggiore \(2015\)](#), and [Itskhoki and Mukhin \(2017\)](#).<sup>7</sup> [Maggiore \(2013\)](#) considers a two country model characterized by an asymmetry in financial intermediation capacity: In his model, one country (US) has a better developed (i.e., less credit constrained) intermediation sector. During global crises, US suffers heavier losses (through wealth transfers to the rest of the world) because of its role as a global insurer, leading to asymmetric international risk sharing.<sup>8</sup> [Maggiore \(2013\)](#) highlights how these effects lead to a “reserve currency paradox”, forcing the US dollar to depreciate in bad times and hence playing against the role of the US dollar as a global safe asset. Our model allows us to look at the reserve currency paradox from a different angle. Specifically, intermediation markups in countries with larger capacity are more sensitive to global conditions. As a result, at times of a global crash, intermediaries in such countries suffer more, their marginal utility spikes, leading to a currency appreciation. Furthermore, we show that US dollar may arise as a global safe haven currency<sup>9</sup> if US monetary policy features a lower amount of uncertainty and/or reacts more aggressively to deteriorating global macroeconomic conditions. In this case, US intermediaries endogenously

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<sup>7</sup>Several papers (see, for example, [Jeanne and Rose \(2002\)](#), [Evans and Lyons \(2002\)](#), [Hau and Rey \(2006\)](#), [Bruno and Shin \(2014\)](#), [Camanho et al. \(2017\)](#)) study the impact of frictions on exchange rates without modelling fundamentals such as exports and imports of multiple goods. Instead, they focus on how the behaviour and incentive structure of intermediaries shapes market outcomes in foreign exchange.

<sup>8</sup>[Kindleberger \(1965\)](#), [Despres et al. \(1966\)](#), [Caballero et al. \(2008\)](#), [Mendoza et al. \(2009\)](#), and [Chien and Naknoi \(2015\)](#) also emphasize differences in financial development across countries as an important source of global imbalances.

<sup>9</sup>That is, a currency that appreciates at a time of a global crisis.

sell more insurance against global crisis states, which in turn makes them suffer more when such a crisis arrives, making the US dollar appreciate in bad states.

[Gabaix and Maggiori \(2015\)](#) develop a general equilibrium model of exchange rates based on the limited risk bearing capacity of financial intermediaries.<sup>10</sup> In their model, intermediaries demand a risk premium for holding currency risk originating in global imbalances. [Gabaix and Maggiori \(2015\)](#) show that this simple intermediation friction has a major impact on equilibrium exchange rates dynamics; in particular, their model is able to rationalize many of the important stylized facts about exchange rates, and link these stylized facts to intermediaries’ balance sheets.

Both in [Gabaix and Maggiori \(2015\)](#) and in our paper, imperfections arise from price pressure effects in the D2C market segment. However, the nature of this price pressure in our model is different from that in [Gabaix and Maggiori \(2015\)](#) and stems from imperfect competition and an endogenous market fragmentation. In particular, in contrast to models with exogenously specified limits to market participation,<sup>11</sup> our model shows how barriers to international trade (intermediation markups) arise endogenously and are determined by forces of supply and demand such as customers’ “reaching for yield” (e.g., through a carry trade) and “flight to safety” whereby customers are attracted by “safe haven” currencies. Finally, in [Gabaix and Maggiori \(2015\)](#) the dynamics of intermediaries’ risk bearing capacity is specified exogenously, while in our model it is endogenous, and is proportional to intermediaries’ net worth. Negative shocks to this net worth occur whenever states against which intermediaries sell a lot of insurance are realized, leading to a redistribution of wealth and affecting state-contingent risk premia and exchange rates dynamics.

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<sup>10</sup>The importance of intermediation frictions for the transmission and the amplification of shocks in domestic markets has been acknowledged in many papers. See, for example, [Holmstrom and Tirole \(1997\)](#), [Bernanke et al. \(1999\)](#), [Gertler and Kiyotaki \(2010\)](#), [He and Krishnamurthy \(2011, 2013, 2014\)](#), [Adrian and Boyarchenko \(2012\)](#), [Brunnermeier and Sannikov \(2014, 2016\)](#), [Adrian et al. \(2014\)](#), [Rampini and Viswanathan \(2015\)](#), [He et al. \(2016b\)](#), [Korinek and Simsek \(2016\)](#), [Piazzesi and Schneider \(2016\)](#), [Bianchi and Bigio \(2016\)](#), [Bigio and Sannikov \(2016\)](#), [Malamud and Schrimpf \(2016\)](#), [Malherbe and McMahon \(2017\)](#), and [Coimbra and Rey \(2017\)](#).

<sup>11</sup>Such as those of [Alvarez et al. \(2002, 2009\)](#) and [Bacchetta and Van Wincoop \(2010\)](#).



[Itskhoki and Mukhin \(2017\)](#) develop a dynamic model similar to that of [Gabaix and Maggiori \(2015\)](#), with exogenous small but persistent shocks to international bond markets. They show that a model with such financial shocks alone is quantitatively consistent with the empirically observed joint dynamics of exchange rates and macro variables.

Our paper is also linked to the literature on the international monetary policy spillovers. For example, [Rey \(2013\)](#) discusses the classical Mundellian trilemma<sup>12</sup>, suggesting that US monetary policy exerts a significant impact even on economies with large financial markets, questioning the conventional wisdom that flexible exchange rates are enough to guarantee monetary autonomy in a world of large capital flows, contrary to the conventional wisdom (see, e.g., [Obstfeld and Taylor \(2004\)](#)). In our model, we show explicitly how monetary policy shocks are transmitted internationally, and how they are linked to local and global intermediation capacities. In particular, we are able to explicitly compute the global “matrix” of monetary transmission (see, [Shin \(2017\)](#) and [Kearns et al. \(2018\)](#)) and show that the impact of monetary shocks in a given country on exchange rates between other countries is proportional to these countries’ relative intermediation capacities.<sup>13</sup>

In our model, intermediation markups charged for insurance against some states of the world can become prohibitively high,<sup>14</sup> making pricing kernel and exchange rates exhibit behaviour reminiscent of “rare disasters” (see, e.g., [Barro \(2006\)](#)). As [Farhi and Gabaix \(2016\)](#) demonstrate, rare disaster risk has a first order impact on equilibrium exchange rates,<sup>15</sup> and can be used to explain a wide array of international asset pricing puzzles. In particular, [Farhi and Gabaix \(2016\)](#) show that a currency that is prone to disaster risk features a high

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<sup>12</sup>The Mundellian trilemma implies that flexible exchange rates are able to perfectly absorb foreign monetary policy shocks in the presence of free capital mobility.

<sup>13</sup>In our model, this result is driven by the fact that balance sheets of (imperfectly competitive) intermediaries play a key role in the international transmission of shocks. See [Miranda-Agrippino and Rey \(2015\)](#) for some supporting evidence.

<sup>14</sup>While in our model the pricing kernel is “tilted” due to intermediary market power, there may be many other mechanisms that shift intermediated prices of insurance against certain states of the world. See, for example, [Malherbe and McMahon \(2017\)](#).

<sup>15</sup>See, also [Farhi et al. \(2015\)](#), [Brunnermeier et al. \(2008\)](#), and [Jurek and Xu \(2014\)](#).

implied volatility in options markets and a significant options skew. Furthermore, [Farhi and Gabaix \(2016\)](#) also show that the most risky currencies (defined as currencies that have the largest exposure to global disaster risk) have a positive correlation with world equity market returns, whereas the least risky currencies have a negative correlation, consistent with the findings of [Lustig et al. \(2011\)](#). Our model is also able to generate similar phenomena, but the disasters arise *endogenously*.

Our paper is also related to the recent work by [Farhi and Maggiori \(2017\)](#). In this paper, they develop a model of the international monetary system and study the role of global safe asset providers in determining the structure of this system.<sup>16</sup> In our model, the nature of the conduct of domestic monetary policy, in particular its responsiveness to domestic and global shocks emerges as a major determinant of the safety properties of local currencies. Our model implies that the status for a given currency of being a globally safe asset is intimately linked to expectations about the future state-contingent conduct of monetary policy. To the best of our knowledge, this implication is unique to our model and is different from other models of currency stabilization, such as, for example, that of [Hassan et al. \(2016\)](#).<sup>17</sup>

Finally, our paper is also related to the recent literature on the breakdown of covered interest parity. See, for example, [Du et al. \(2016\)](#), [Avdjiev et al. \(2016\)](#), [Borio et al. \(2016\)](#), and [Rime et al. \(2017\)](#). Several papers derive CIP deviations using models with different forms of limits-to-arbitrage. For example, [Amador et al. \(2017\)](#) show how CIP deviations arise in a small open economy at the zero lower bound; [Ivashina et al. \(2015\)](#) and [Liao \(2016\)](#) highlight global banks' demand for dollar funding as drivers of CIP deviations;<sup>18</sup> [Hebert \(2017\)](#) shows how the cross-section of CIP deviations can be used to recover intermediaries'

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<sup>16</sup>See also [He et al. \(2016a\)](#), who investigate the emergence of endogenous safe assets in a global games framework.

<sup>17</sup>[Hassan et al. \(2016\)](#) show how state-contingent monetary/stabilization policies impact the risk properties of exchange rates.

<sup>18</sup>See, also, [Aldasoro et al. \(2017\)](#) who provide evidence that Japanese banks, which are known to have a particularly high demand for US dollar funding, face significant markups when accessing dollar funding markets, consistent with the mechanism highlighted in our model.

financial constraints; Andersen et al. (2017) show how seemingly riskless arbitrage (including CIP) may not be economically viable due to dealers' funding value adjustment; and Jiang et al. (2018) assign a key role to the special role of US Treasury securities as collateral. To the best of our knowledge, our model is the first multi-country macroeconomic general equilibrium model that generates a breakdown of CIP endogenously, through price pressure effects in imperfect international financial markets. In particular, we are able to shed some light on the macroeconomic origins of CIP deviations, their signs and differences across countries.

## 2 The Model

### 2.1 Agents, Preferences, and Consumption

We consider a standard, international multiple goods monetary economy with intra-temporal<sup>19</sup> cash-in-advance constraints, as in Lucas (1982). Time is discrete,  $t = 0, 1, \dots, T$ , and the information structure is characterized by a probability space  $(\Omega, P)$  equipped with a filtration  $(\mathcal{F}_t)_{t \geq 0}$ . There are  $N$  countries, indexed by  $i = 1, \dots, N$ . Country  $i$  produces a tradable good, also indexed by  $i$ . Country  $i$  tradable good is produced by an endowment process  $X_{i,t}$ ,  $i = 1, \dots, N$ ,  $t \geq 0$ . The government of country  $i$  controls the supply of domestic, country  $i$  currency,  $\mathcal{M}_{i,t}$ .

Each country is populated by two classes of agents,  $I$ -agents (intermediaries, or, dealers) and  $H$ -agents (households, or, customers) that have identical,<sup>20</sup> time-varying, stochastic time discount factors  $\Psi_{i,t}$ ,  $i = 1, \dots, N$ . We assume that all agents derive utility from consumption  $C_{i,t}^{bundle}$  of a country-specific bundle of tradable goods. All agents are endowed

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<sup>19</sup>That is, agents only need to hold cash within the period for consumption needs, and do not store cash inter-temporally. For example, this can be achieved if intermediaries deposit their cash holdings overnight with the central bank, and the bank pays interest on these cash, equal to the equilibrium nominal rate. This interest is then simply a part of the total cash rebates  $\mathcal{M}_{i,t} - \mathcal{M}_{i,t-1}$ .

<sup>20</sup>In the Appendix, we report results for the case when households and intermediaries have different discount factors.

with standard, inter-temporal, logarithmic preferences

$$E \left[ \sum_{t=0}^T \Psi_{i,t} \log C_{i,t}^{bundle} \right], \quad i = 1, \dots, N,$$

where

$$C_{i,t}^{bundle,I,H} = \prod_{k=1}^N (C_{i,k,t}^{I,H})^{\theta_{i,k}}, \quad i = 1, \dots, N$$

is the country-specific tradable goods consumption bundle. Here,  $C_{i,k,t}^{I,H}$ ,  $k = 1, \dots, N$  is the time- $t$  consumption of country- $k$  tradable good in country  $i$  by the corresponding agents' class  $I, H$ . Without loss of generality, we normalize these preference parameters so that

$$\sum_k \theta_{i,k} = 1$$

for all  $i = 1, \dots, N$ .

Denote by  $P_{i,k,t}$  the nominal price of good  $k$  in country  $i$ , in the units of the local currency. We also denote by  $\mathcal{E}_{i,t}$  the US dollar price of the currency of country  $i$ , that is, whenever  $\mathcal{E}_{i,t}$  goes up, the local currency of country  $i$  appreciates against the US dollar. Below, we will always use currency of country 1 (US dollars) as the reference currency and use \$ to denote the corresponding economic variables. Since our focus is on financial market frictions, we abstract from frictions in international goods markets and assume that purchasing power parity (the law of one price) always holds.<sup>21</sup> In this case, nominal goods prices in local currencies satisfy

$$\mathcal{E}_{i,t} P_{i,k,t} = \mathcal{E}_{k,t} P_{k,k,t}, \quad i, k = 1, \dots, N,$$

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<sup>21</sup>Similarly to [Itskhoki and Mukhin \(2017\)](#), we could also introduce shocks to the law of one price and study its impact on exchange rates.

and hence the cash-in-advance constraint implies that the total nominal expenditures for country  $k$  tradable good's endowment  $X_{k,t}$  always equals money supply:

$$P_{k,k,t} X_{k,t} = \mathcal{M}_{k,t}, \quad k = 1, \dots, N, \quad t \geq 0. \quad (1)$$

The following lemma characterizes the optimal choice of money-consumption bundles.

**Lemma 1** *Given the total nominal expenditure in the units of currency  $i$ ,*

$$C_{i,t}^{I,H} \equiv \sum_{k=1}^N P_{i,k,t} C_{i,k,t}^{I,H},$$

*the optimal consumption bundle of the respective agent class is given by*

$$C_{i,k,t}^{I,H} = C_{i,t}^{I,H} P_{i,k,t}^{-1} \theta_{i,k}, \quad i, k = 1, \dots, N.$$

## 2.2 Financial Market Structure

We assume that all class  $I$  agents (intermediaries) from each country  $i = 1, \dots, N$  have a direct access to a frictionless, complete, centralized, international dealer-to-dealer (D2D) market. We interpret these agents as specialists who possess a technology that allows them to issue and trade general state-contingent claims (a full set of Arrow securities) with other agents. Since markets are complete, the prices of all financial securities traded in the inter-dealer market can be encoded in a single, international US dollar nominal pricing kernel  $M_{\$,t,t+1}^I$  quoted in the units of currency 1, so that the time- $t$  US dollar price  $q_t$  of a state-contingent claim with a dollar payoff  $Y_{t+1}$  is given by

$$q_t = E_t[M_{\$,t,t+1}^I Y_{t+1}].$$

In the sequel, we will refer to  $M_{\$,t,t+1}^I$  as the (US dollar) dealer-to-dealer (D2D) pricing kernel. In stark contrast to class- $I$  agents, class- $H$  agents (henceforth, customers) of a given country  $i$  do not have a direct access to the inter-dealer market, except for the possibility to trade the claim on their endowment  $X_{i,t}$  (the stock index of country  $i$ ) as well as one-period nominal risk-free bonds of their respective country. Customers willing to trade any other financial instrument need to contact an intermediary (an intermediation firm) and bargain over the counter in a dealer-to-customer (D2C) market. This means that customers can borrow or lend in their local currency at prevailing market rates, but those that wish to borrow or lend in a foreign currency need to do so through intermediaries.<sup>22</sup>

Following [He and Krishnamurthy \(2013\)](#), we assume class- $I$  agents are specialists who run intermediation firms. The objective of such a firm is to maximize the firm value (that is, the present discounted value of intermediation markups) under the D2D pricing kernel. Since markets are complete, the risk neutral firms' objective coincides with that of the risk averse specialists who run it: Indeed, both the firm and the specialist's objective is to maximize the present value of revenues under the unique pricing kernel. Therefore, in the following we identify class- $I$  agents with the intermediation firm they run and we call them intermediaries.<sup>23</sup> We formalize the details of the bargaining protocol in the following assumption (see [Figure 1](#) below for a graphical description).

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<sup>22</sup>In our model, trading foreign stocks can also be done only through intermediaries. This assumption allows us to capture the fact that trading and owning foreign stocks often involves significant amounts of intermediation. For example, a US investor can invest in foreign stocks through American Depositary Receipts (ADRs). But, in reality this transaction goes through an intermediary (a custodian bank) who is in charge of actually holding the ADR. The custodian charges intermediation fees for maintaining the ADR records, collecting the dividends paid out by the foreign issuer, converting it into US dollars and depositing into the stockholder's account. Thus, effectively, ADR is an OTC contract between the investor and the custodian bank. Similarly, short selling a stock (both local and foreign) always involves intermediation, whereby the short seller has to go to an intermediary who then needs to locate a stock owner to borrow the stock. See, e.g., [Duffie et al. \(2005\)](#).

<sup>23</sup>For simplicity, we assume that specialists are the only shareholders of intermediaries and hence markups are not rebated back to customers: By assumption, customers (class- $H$  agents) can only freely trade claims on their wealth and short term bonds. This assumption is made for simplicity and can be relaxed. Allowing customers to freely trade intermediary stocks would add another Lagrange multiplier to the shadow costs of intermediation and hence would complicate the analysis.

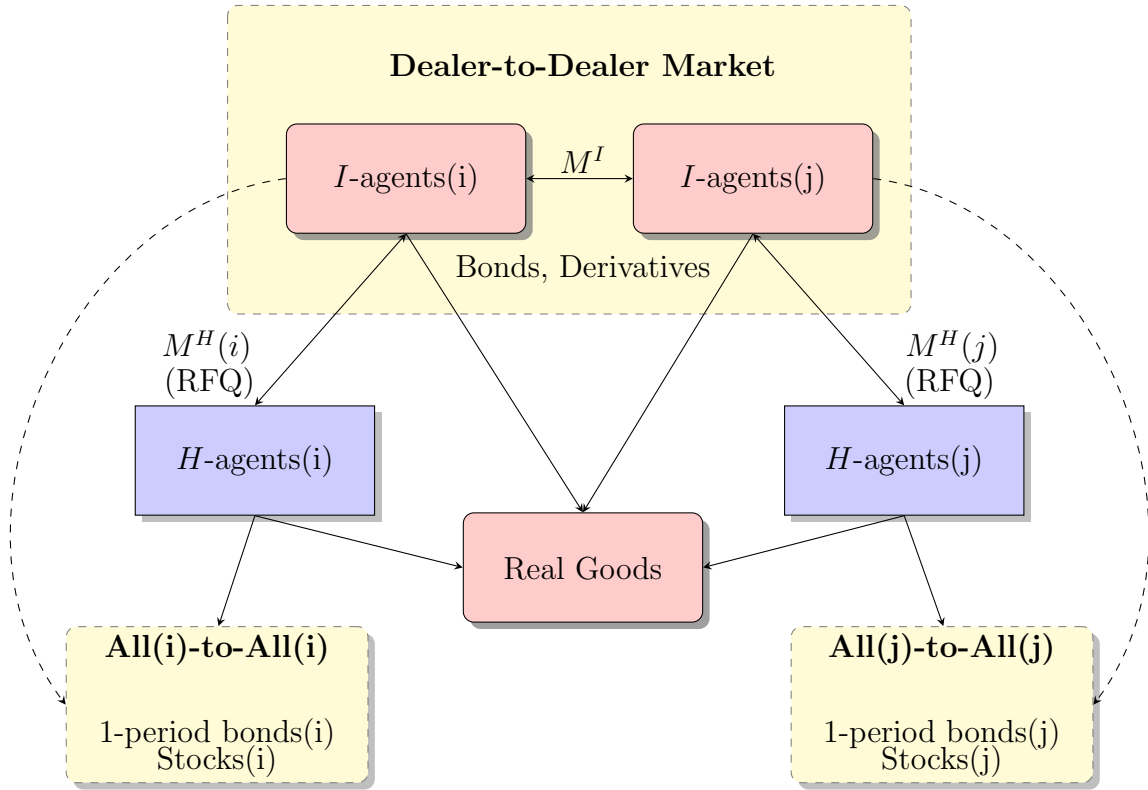
**Assumption 1** *In the beginning of each period  $t$ , each customer of country  $i$  is matched with an intermediary of the same country and requests quotes for prices of all one-period-ahead state-contingent claims.<sup>24</sup> The intermediary quotes a one period ahead country-specific D2C pricing kernel  $M_{i,t,t+1}^H$  in the local currency and has full bargaining power in choosing  $M_{i,t,t+1}^H$  due to search frictions: If the customer rejects the offer, he can trade country  $i$  endowment claims and country  $i$  one-period risk free bonds in the country  $i$  centralized market with other country  $i$  investors, and then has to wait one more period until he is matched with another intermediary. The quotes are binding: After receiving the quote, the customer chooses an optimal bundle of state-contingent claims, and the intermediary sells this bundle to the customer at the quoted prices.*

The key mechanisms in our model depend crucially on the ability of intermediaries to extract rents. The assumption of monopolistic competition is made for tractability reasons and can be relaxed; for example, our results can be easily adjusted to allow for a different bargaining protocol with a bargaining power below one, such as the Nash protocol that is commonly used in the literature on OTC markets. See, [Duffie et al. \(2005\)](#), [Duffie et al. \(2007\)](#)) and [Lagos and Rocheteau \(2009\)](#).<sup>25</sup> However, some papers (see, for example, [Petersen and Rajan \(1995\)](#)) argue that monopolistic competition in the intermediation sector is a closer approximation to reality due to switching and relationship costs. See, also, [Sharpe \(1997\)](#), [Kim et al. \(2003\)](#), [Bolton et al. \(2016\)](#), [Brunnermeier and Koby \(2016\)](#), [Duffie and Krishnamurthy \(2016\)](#), and [Acharya and Plantin \(2016\)](#).

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<sup>24</sup>The assumption of trading only one period claims with intermediaries is standard in the literature. As [Brunnermeier and Koby \(2016\)](#) argue, this is without loss of generality if old contracts are indexed on contemporaneous economic conditions.

<sup>25</sup>The new regulatory environment (based on the Dodd-Frank act) is designed to move bilateral relationship trading to electronic platforms. For example, trading of standardized interest rate swaps in the US has to a large extent moved to so-called swap execution facilities (SEFs). An all-to-all market such as in equities markets remains a distant reality, though. Most D2C transaction are executed via a request for quote (RFQ) protocol, which is equivalent to an electronic form of OTC trading. The original two-tier market structure thus shows remarkable persistence, with a D2D segment at the core of the market, as in our model. The same is true for fixed income and foreign exchange markets. See [Collin-Dufresne et al. \(2016\)](#), [Bech et al. \(2016\)](#), and [Moore et al. \(2016\)](#).



**Figure 1:** Graphical description of market structure in our model for the two country case (country  $i$  and country  $j$ ). RFQ denotes the request for quote protocol commonly used in D2C segments of OTC markets.



## 2.3 D2C Bargaining and Markups

Assumption 1 implies that we can formulate the bargaining problem in terms of the local currency nominal D2C state prices  $M_{i,t,t+1}^H$  quoted by the country  $i$  intermediary to a country  $i$  customer.<sup>26</sup> Even though customers can only trade one-period claims, market completeness implies that agents can effectively replicate any stream of nominal expenditures in the local currency,  $(C_t^H)_{t \geq 0}$ , with the prices of  $t$ -period ahead Arrow-Debreu claims given through the nominal local currency-denominated stochastic discount factor  $M_{i,0,t}^H = M_{i,0,1}^H M_{i,1,2}^H \cdots M_{i,t-1,t}^H$ . The multi-period D2D dollar pricing kernel is defined similarly:  $M_{\$,0,t}^I = M_{\$,0,1}^I M_{\$,1,2}^I \cdots M_{\$,t-1,t}^I$ . By no arbitrage, the country  $i$  D2D pricing kernel denominated in local currency is linked to dollar D2D pricing kernel through the identity

$$M_{i,0,t}^I = M_{\$,0,t}^I \mathcal{E}_{i,t} / \mathcal{E}_{i,0}. \quad (2)$$

We will use  $r_{i,t}$  to denote the short term nominal interest rate, and we will let  $S_{i,t}$  denote the nominal present value of the total endowment,  $X_{i,t}$ , of the local, country- $i$  good. By the cash-in-advance constraint (see (1)),  $S_{i,t}$  also coincides with the present value of total money supply,  $\mathcal{M}_{i,\tau}$ ,  $\tau \geq t$ . Hereafter, we interpret this claim as the country- $i$  stock index and call it the local stock price. By assumption, local customers can freely trade the endowment claim as well as one-period nominal risk-free bonds. This means that the intermediary has to quote fair prices for both instruments: Otherwise, customers would immediately arbitrage away the differences in the quoted and the inter-dealer rate, leading to unbounded losses for the intermediary. Formally, this means that the D2C pricing kernel  $M_{i,t,t+1}^H$  quoted by the intermediary has to satisfy two constraints relating the short term rate  $r_{i,t}$  and the stock

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<sup>26</sup>Hebert (2017) investigates a model with a form of market segmentation that is similar to that assumed in our paper. Namely, Hebert (2017) considers an incomplete market model in which intermediaries can trade a full set of state-contingent claims with each other in the D2D market, while households are constrained in the set of assets they can trade with each other and with intermediaries, who are facing convex portfolio constraints. As a result of this segmentation, Hebert's model also features two pricing kernels, as well as deviations from the law of one price.

price,  $S_{i,t}$ , in the two market segments:

$$e^{-r_{i,t}} \equiv E_t[M_{i,t,t+1}^H] = E_t[M_{i,t,t+1}^I] \quad (3)$$

$$S_{i,t} \equiv \mathcal{M}_{i,t} + E_t[M_{i,t,t+1}^H S_{i,t+1}] = \mathcal{M}_{i,t} + E_t[M_{i,t,t+1}^I S_{i,t+1}]. \quad (4)$$

The first equation simply requires that the one period domestic nominal bond trades at the same price in the two market segments. The second one is more subtle: It means that the domestic stock market is fairly priced under both pricing kernels. We will also make the following assumption.

**Assumption 2** *We assume that class I and class H agents in country  $i$  are endowed with the respective shares  $\alpha_i$  and  $1 - \alpha_i$  of the total endowment of the country  $i$  tradable good. At time zero, intermediaries pay a cost  $\bar{K}_{i,0}$  to customers to set up intermediation firms. The monetary authority controls money supply through direct rebates to intermediaries.<sup>27</sup> We also use  $\mathcal{N}_{i,t+1} \equiv \mathcal{M}_{i,t+1}/\mathcal{M}_{i,t}$  and  $\mathcal{N}_{i,t,\tau} \equiv \mathcal{M}_{i,\tau}/\mathcal{M}_{i,t}$  to denote the growth in money supply, and we will use the normalization  $E_t[\mathcal{N}_{i,t+1}^{-1}] = 1 = \mathcal{M}_{i,0}$  for all  $i = 1, \dots, N$ ,  $t \geq 0$ .*

By Assumption 2, customers' time zero nominal net worth is given by  $W_{i,0}^H = (1 - \alpha_i)S_{i,0} + \bar{K}_{i,0}$ . Since markets are complete, customers can use trading in the D2C market to attain any state contingent consumption expenditures profile  $(C_{i,t}^H)_{t \geq 0}$  in the local currency satisfying the inter-temporal budget constraint:

$$E \left[ \sum_{t=0}^T C_{i,t}^H M_{i,0,t}^H \right] = W_{i,0}^H.$$

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<sup>27</sup>For example, as Brunnermeier and Sannikov (2016) argue, controlling the rate on the central bank reserves is effectively equivalent to controlling the supply of central bank money, whereby interest payments on reserves are equivalent to direct money rebates to intermediaries. Note, however, that market segmentation implies that the distribution of money holdings has real effects in our model and hence cannot be neglected.

Let

$$D_{i,t} \equiv E_t \left[ \sum_{\tau=0}^{T-t} \Psi_{i,t,t+\tau} \right], \quad (5)$$

where we have defined

$$\Psi_{i,t,t+\tau} \equiv \frac{\Psi_{i,t+\tau}}{\Psi_{i,t}}$$

to be the multi-period discount factors between time  $t$  and time  $t + \tau$ .<sup>28</sup> That is,  $D_{i,t}$  is the expected discount factor for the whole future consumption stream. We also define

$$D_{i,t,\tau} \equiv \frac{D_{i,t+\tau}}{D_{i,t}}, \quad i = 1, \dots, N.$$

We will also use  $W_{i,t}^{H,I}$ ,  $i = 1, \dots, N$ , to denote the nominal wealth of the corresponding agents' class. Standard results imply that the following is true.

**Lemma 2** *Country  $i$  customers' nominal consumption and wealth dynamics are given by*

$$C_{i,t}^H = W_{i,0}^H \frac{\Psi_{i,t} (M_{i,0,t}^H)^{-1}}{D_{i,0}^H} \quad (6)$$

and<sup>29</sup>

$$\frac{W_{i,t}^H}{W_{i,t-1}^H} = (M_{i,t-1,t}^H)^{-1} \Psi_{i,t-1,t} D_{i,t-1,t}, \quad i = 1, \dots, N.$$

By Assumption 2, class- $I$  agents are initially endowed with  $\alpha_i$  shares of the claim on the aggregate endowment plus the money rebates from the government. In addition, they own the intermediation firms that generate a nominal income flow  $\mathcal{I}_{i,t}$  in the local currency from

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<sup>28</sup>If we define  $\beta_{i,t+1} = -\log \Psi_{i,t,t+1}$  to be the one-period discount rate, one can rewrite  $\Psi_{i,t,t+\tau} = e^{-\sum_{s=1}^{\tau} \beta_{i,t+s}}$ .

<sup>29</sup>Importantly, our model features a non-constant consumption/wealth ratio, proportional to  $D_{i,t}$ .

intermediation mark-ups. Hence, their nominal net worth is given by

$$W_{i,0}^I = -\bar{K}_{i,0} + \alpha_i S_{i,0} + E \left[ \sum_{t=0}^T M_{i,0,t}^I (\mathcal{I}_{i,t} + (\mathcal{M}_{i,t} - \mathcal{M}_{i,t-1})) \right],$$

where  $\bar{K}_{i,0}$  is the time zero (entry) cost of setting up an intermediation firm.<sup>30</sup>

The same argument as that for (6) implies that an intermediary's optimal consumption expenditures in the local currency are given by

$$C_{i,t}^I = W_{i,0}^I \frac{\Psi_{i,t} (M_{i,0,t}^I)^{-1}}{D_{i,0}^I}. \quad (7)$$

Let us now consider the bargaining problem between a customer and an intermediary. At time  $t$ , a country  $i$  customer with the nominal wealth  $W_{i,t}^H$  gets matched with an intermediary who quotes him a one period ahead pricing kernel  $M_{i,t,t+1}^H$  in the local currency. Given this quote, the customer decides how to optimally finance his future excess consumption,  $C_{i,t+1}^H$  through a portfolio of the risk-free bond and the stock to be traded in the centralized market, as well as an OTC contract with a state-contingent payoff that he buys in the D2D market. Due to the no-arbitrage constraints for the local stock and bond markets (see Assumption 1), customers are in fact indifferent between trading the stock and bond in the D2D and the D2C market. Hence, without loss of generality we can assume that they directly trade bonds and stocks with intermediaries. Thus, the agent is simply buying the claim on his future wealth,  $W_{i,t+1}^H$ , from the intermediary, so that current consumption is equal to the difference

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<sup>30</sup>As we explain above, we assume that this nominal cost is immediately transferred to customers at time zero. The assumption that the cost is only incurred at time zero is made for convenience and can be relaxed. These costs will play no role in the subsequent analysis. One could potentially use them to endogenize the size of the intermediation sector as well as to study the impact of regulations on the endogenous size of intermediation sector and markups. Importantly, making these costs sufficiently large, we can make intermediary net worth,  $W_{i,0}^I$ , arbitrarily small. They also allow us to make an important distinction between the size of markups and the actual profitability of the intermediation sector: While the markups (i.e., the spread between the D2C and the D2D pricing kernels) might be high, the actual profit margins might be quite low.

between the current wealth and the D2C price of the claim on future wealth:

$$C_{i,t}^H = W_{i,t}^H - E_t[M_{i,t,t+1}^H W_{i,t+1}^H].$$

The customers' problem is thus to solve for the optimal interplay between today's consumption  $C_{i,t}^H$  and tomorrow's wealth. The following is a direct consequence of Lemma 2.

**Lemma 3** *Optimal demand of a country- $i$  customer in the D2C market is explicitly given by*

$$W_{i,t+1}^H(M_{i,t,t+1}^H) = \frac{W_{i,0}^H}{D_{i,0}^H} \Psi_{i,t,t+1} D_{i,t+1} \Psi_{i,0,t} (M_{i,0,t}^H)^{-1} (M_{i,t,t+1}^H)^{-1}.$$

The intuition behind Lemma 3 is straightforward: A log utility maximizing agent always consumes inversely proportionally to state prices (Equation (6)). Furthermore, his decision to allocate wealth across states is driven by the product of the discount factor  $\Psi_{i,t,t+1}$  and the expected discount factor  $D_{i,t+1}$ ; the latter determines the value of the total future stream of consumption in a given state.

The time  $t$  value of the claim on  $W_{i,t+1}^H$  for the intermediary is given by  $E_t[M_{i,t,t+1}^I W_{i,t+1}^H]$ , and intermediary's objective is to maximize the total markup

$$\mathcal{I}_{i,t} = E_t[M_{i,t,t+1}^H W_{i,t+1}^H] - E_t[M_{i,t,t+1}^I W_{i,t+1}^H] = E_t[(M_{i,t,t+1}^H - M_{i,t,t+1}^I) W_{i,t+1}^H]$$

given by the difference between the value of the claim  $W_{i,t+1}^H$  under the D2C and the D2D pricing kernels.<sup>31</sup> By Lemma 3, the markup maximization problem of the intermediary takes

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<sup>31</sup>Indeed, since intermediaries have access to complete D2D markets, their objective is to maximize the present value of cash flows in the D2C market under the D2D pricing kernel. Those cash flows are given by  $E_t[M_{i,t,t+1}^I W_{i,t+1}^H]$  at time  $t$  and by  $-W_{i,t+1}^H$  at time  $t+1$ , and the present value is given by  $\mathcal{I}_{i,t}$ .

the form

$$\begin{aligned} & \max_{M_{i,t,t+1}^H > 0} E_t[(M_{i,t,t+1}^H - M_{i,t,t+1}^I) W_{i,t+1}^H (M_{i,t,t+1}^H)] \\ &= \frac{W_{i,0}^H}{D_{i,0}^H} \Psi_{i,0,t} (M_{i,0,t}^H)^{-1} \max_{M_{i,t,t+1}^H > 0} E_t[(M_{i,t,t+1}^H - M_{i,t,t+1}^I) \Psi_{i,t,t+1} D_{i,t,t+1} (M_{i,t,t+1}^H)^{-1}] \end{aligned} \quad (8)$$

under the constraints (3)-(4). Denoting by  $\mu_{i,t}$  and  $\lambda_{i,t}$  the Lagrange multipliers for constraints (3) and (4) respectively, and writing down the first order conditions for (8), we get

$$M_{i,t,t+1}^I \Psi_{i,t,t+1} D_{i,t,t+1} (M_{i,t,t+1}^H)^{-2} = \lambda_{i,t} (S_{i,t+1}/S_{i,t}) + \mu_{i,t}. \quad (9)$$

The intuition behind (9) is as follows: The marginal gain of selling insurance against a state  $x$  is given by the product of the D2D price  $M^I(x)$  and the sensitivity of customer's consumption to the price  $M^H(x)$ . Since customers have log utility, this sensitivity is given by  $-\Psi_{i,t,t+1} D_{i,t,t+1} (M_{i,t,t+1}^H)^{-2}$ . At the optimum, this marginal gain is equal to the *state-contingent* shadow cost of constraints (3)-(4), given by  $\lambda_{i,t} (S_{i,t+1}/S_{i,t}) + \mu_{i,t}$ . The solution to (9) is reported in the following proposition.

**Proposition 4** *The optimal pricing kernel quoted by the intermediary is given by*

$$M_{i,t,t+1}^H = \frac{(\Psi_{i,t,t+1} D_{i,t,t+1})^{1/2} (M_{i,t,t+1}^I)^{1/2}}{(\lambda_{i,t} (S_{i,t+1}/S_{i,t}) + \mu_{i,t})^{1/2}}, \quad (10)$$

where the Lagrange multipliers  $\lambda_{i,t}, \mu_{i,t} \in \mathbb{R}$  are determined by the conditions

$$\begin{aligned} E_t \left[ \frac{(\Psi_{i,t,t+1} D_{i,t,t+1})^{1/2} (M_{i,t,t+1}^I)^{1/2}}{(\lambda_{i,t} (S_{i,t+1}/S_{i,t}) + \mu_{i,t})^{1/2}} \right] &= E_t[M_{i,t,t+1}^I]; \\ E_t \left[ \frac{(\Psi_{i,t,t+1} D_{i,t,t+1})^{1/2} (M_{i,t,t+1}^I)^{1/2} S_{i,t+1}}{(\lambda_{i,t} (S_{i,t+1}/S_{i,t}) + \mu_{i,t})^{1/2}} \right] &= E_t[M_{i,t,t+1}^I S_{i,t+1}]. \end{aligned}$$

Proposition 4 is key to the subsequent analysis. It shows how the bargaining friction and

the ability of intermediaries to charge state-contingent markups affects asset prices and, as a result, distorts equilibrium allocations.<sup>32</sup>

The signs of  $\lambda_{i,t}$ ,  $\mu_{i,t}$  will play a very important role in the subsequent analysis. Since these quantities are Lagrange multipliers of constraints (3)-(4), their signs are determined by the “direction” in which these constraints are binding. Consider first the Lagrange multiplier  $\mu_{i,t}$  of the constraint (3). One can equivalently interpret (3) as a pair of inequality constraints

$$\begin{aligned} E_t[M_{i,t,t+1}^H] &\geq (1 - \varepsilon)E_t[M_{i,t,t+1}^I] \\ E_t[M_{i,t,t+1}^H] &\leq (1 + \varepsilon)E_t[M_{i,t,t+1}^I], \end{aligned} \tag{11}$$

where the parameter  $\varepsilon$  (determining the corridor inside which the intermediary can quote rates) is arbitrarily small.

The economic intuition behind these constraints is as follows. If customers would like to invest into risk free assets<sup>33</sup>, the intermediary will try to push the nominal rate  $e^{r_{i,t}} = 1/E_t[M_{i,t,t+1}^H]$  all the way down to its lower bound, determined by the D2D market rate  $1/E_t[M_{i,t,t+1}^I]$ , and hence the constraint  $E_t[M_{i,t,t+1}^H] \leq E_t[M_{i,t,t+1}^I]$  will be binding; in this case, standard Kuhn-Tucker conditions imply that  $\mu_{i,t} > 0$ .

By contrast, if customers find it optimal to borrow from intermediaries, the latter will try to push the offered rate all the way up to its upper bound, determined by the D2D market rate  $1/E_t[M_{i,t,t+1}^I]$ , and hence the constraint  $E_t[M_{i,t,t+1}^H] \geq E_t[M_{i,t,t+1}^I]$  will be binding; in this case, standard Kuhn-Tucker conditions imply that  $\mu_{i,t} < 0$ . Similar intuition applies to (4).

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<sup>32</sup> Another important consequence of Proposition 4 is the break-down of money neutrality. The mechanism underlying this non-neutrality is related to the Fisher debt deflation theory, whereby unexpected monetary shocks serve as a channel for redistributing wealth between customers and intermediaries. See [Malamud and Schrimpf \(2016\)](#) for details.

<sup>33</sup> Intermediaries could provide access to (nearly) risk free assets through private money creation: for example, through bank deposits and money market funds. We abstract from such private money creation in our model. See, [Brunnermeier and Sannikov \(2016\)](#) for a model featuring an impact of such private money creation on monetary policy passthrough.

## 2.4 The Fundamental Markup Equation

We assume that domestic stocks and bonds are not sufficient to span the consumption profile desired by customers.<sup>34</sup> The key role of intermediaries in our model is then to offer customers (risky) alternatives such as foreign bonds or other, more complex instruments. Customers' demand for such securities determines the size and the sign of security's markups (i.e., the spread between the price of the security in the D2C and the D2D market). For a foreign risky security with nominal payoff  $X_{t+1}$  in the local currency, the markup is given by

$$\mathcal{U}_{i,t}(X_{t+1}) \equiv E_t[M_{i,t,t+1}^H X_{t+1}] - E_t[M_{i,t,t+1}^I X_{t+1}].$$

Define

$$\Gamma_{i,t,t+1} \equiv \frac{M_{i,t,t+1}^H}{M_{i,t,t+1}^I} = \frac{(\Psi_{i,t,t+1} D_{i,t,t+1})/M_{i,t,t+1}^H}{\lambda_{i,t}(S_{i,t+1}/S_{i,t}) + \mu_{i,t}} \quad (12)$$

to be the *state-contingent markup*. Denote by  $\widetilde{\text{Cov}}_t^{I,i}$  the covariance under the D2D risk neutral measure in country  $i$ . Using (3), we arrive at the following result.

**Proposition 5** [*The fundamental markup equation*] *Intermediation markups are given by*

$$\mathcal{U}_{i,t}(X_{t+1}) = e^{-r_{i,t}} \widetilde{\text{Cov}}_t^{I,i}(\Gamma_{i,t,t+1}, X_{t+1}). \quad (13)$$

Equation (13) is a *fundamental markup equation*, akin to the fundamental equation of asset pricing that characterizes risk premia through covariance with the stochastic discount factor. It is very intuitive: Assets that payoff in states with high intermediation costs  $\Gamma_{i,t,t+1}$  trade at high markups. It is based on the *risk-based* approach to markups, whereby the risk properties of a security's payoff together with the price pressure created by customers' demand in the D2C market are key determinants of the magnitude of markups.

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<sup>34</sup>For this to be true, we need sufficiently many (at least three) states at each node of the event tree.



The fundamental markup equation is very general and can be used to study intermediation spreads in a large variety of markets, such as, e.g., interest rate derivatives (swaps, swaptions), the bond-CDS basis, and foreign exchange derivatives, all of which trade OTC. Here, we will apply equation (13) to investigate how intermediation frictions impact spreads in international borrowing/lending markets.

Indeed, in the real world, customers in a foreign country often do not have access to direct dollar borrowing and lending.<sup>35</sup> Instead, they have obtain US dollars through intermediaries by borrowing funds in the local currency and then swapping their position into dollar.<sup>36</sup> In a fictitious perfect market, the forces of arbitrage imply that the corresponding FX swap-implied dollar rate should be equal to the dollar rate when directly borrowing funds in US money markets. This arbitrage relationship is known as the covered interest parity (CIP) condition. However, a growing empirical literature (see, for example, Du et al. (2016), Avdjiev et al. (2016), Borio et al. (2016), and Rime et al. (2017)) provides strong evidence for large and persistent CIP deviations across a multitude of currencies.

In our model, market fragmentation naturally leads to a violation of the CIP relationship because customers willing to enter the FX swap position need to do this through intermediaries in an OTC market with non-competitive prices. The FX swap rate quoted by the intermediary will contain a ‘markup’. Our goal here is to study the potential macroeconomic drivers of such deviations. Recall that  $\mathcal{E}_{i,t}$  is the dollar price of country  $i$  currency, and hence  $\mathcal{E}_{i,t}^{-1}$  is the currency- $i$  price of the dollar. For notational convenience, throughout this section, we will denote  $\$_{i,t+1} = \mathcal{E}_{i,t+1}^{-1}$ . We will then need the following definition.

**Definition 6** *We denote by  $r_{\$,t}^{H,i}$  the “synthetic” nominal US dollar interest rate quoted by*

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<sup>35</sup>This also often applies to major foreign regional banks, which are considered as lower-tier in international capital markets and treated as customers by the main global dealer banks.

<sup>36</sup>The global FX swap markets are highly concentrated, with trading dominated by a handful of dealers. See, for example, Moore et al. (2016).

intermediaries to customers of country  $i$ . By definition,

$$e^{-r_{\$,t}^{H,i}} = E_t \left[ M_{i,t,t+1}^H \frac{\$_{i,t+1}}{\$_{i,t}} \right].$$

We also define the cross-currency basis against the US dollar as

$$\text{Basis}_{i,t}^{\$} = e^{-r_{\$,t}} - e^{-r_{\$,t}^{H,i}} \approx r_{\$,t}^{H,i} - r_{\$,t}.$$

That is, the basis (as defined here) is given by the difference between the synthetic US dollar rate  $r_{\$,t}^{H,i}$  that country  $i$  customers can back out from intermediaries' quotes in the FX swap market and the direct dollar rate  $r_{\$,t}$ , is generally non-zero.

It is important to note that the CIP relation holds separately in the D2C and the D2D markets, but not across the two markets. That is, country  $i$  customers receive a quote  $r_{\$,t}^{H,i} = -\log E_t \left[ M_{i,t,t+1}^H \frac{\$_{i,t+1}}{\$_{i,t}} \right]$ , for the Dollar rate, as well as a quote  $f_{\$,t}^{H,i} = \log \frac{E_t [M_{i,t,t+1}^H \$_{i,t+1}]}{E_t [M_{i,t,t+1}^H]}$  for the Dollar forward rate. Both are in the D2C market and satisfy the no-arbitrage CIP relation:  $r_{\$,t}^{H,i} = \log \$_{i,t} - f_{\$,t}^{H,i} + r_{i,t}$ . At the same time, the same relation holds also in the D2D market:  $r_{\$,t} = \log \$_{i,t} - f_{\$,t}^{I,i} + r_{i,t}$ . Thus, the Basis is given approximately by

$$\text{Basis}_{i,t}^{\$} \approx r_{\$,t}^{H,i} - r_{\$,t} = (\log \$_{i,t} - f_{\$,t}^{H,i} + r_{i,t}) - r_{\$,t} = f_{\$,t}^{H,i} - f_{\$,t}^{I,i} \quad (14)$$

and arises exclusively from the difference in the forward rates,  $f_{\$,t}^{H,i} - f_{\$,t}^{I,i}$ , and hence we can interpret  $r_{\$,t}^{H,i}$  as the forward-implied rate.

From the point of view of customers in country  $i$ , a dollar bond<sup>37</sup> is a risky security with the nominal payoff (in country  $i$  currency) given by  $X_{t+1} = \frac{\$_{i,t+1}}{\$_{i,t}}$ . Therefore, by Proposition

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<sup>37</sup>That is, a pure discount zero coupon bond.

5,

$$Basis_{i,t}^{\$} = -e^{-r_{i,t}} \widetilde{\text{Cov}}_t^{I,i} \left( \Gamma_{i,t,t+1}, \frac{\$_{i,t+1}}{\$_{i,t}} \right). \quad (15)$$

Lemma 2 implies that the state-contingent markup (12) is directly linked to the consumption and wealth ratios of customers and intermediaries: Since wealth is inversely proportional to the pricing kernel, we get

$$\Gamma_{i,t,t+1} = \frac{W_{i,t+1}^I / W_{i,t}^I}{W_{i,t+1}^H / W_{i,t}^H},$$

and hence we can rewrite (15) as

$$Basis_{i,t}^{\$} = -e^{-r_{i,t}} \frac{W_{i,t}^H}{W_{i,t}^I} \widetilde{\text{Cov}}_t^{I,i} \left( \frac{W_{i,t+1}^I}{W_{i,t+1}^H}, \frac{\$_{i,t+1}}{\$_{i,t}} \right). \quad (16)$$

Formula (16) shows that the basis is largely determined by two forces: the current net worth ratio,  $\frac{W_{i,t}^H}{W_{i,t}^I}$ ; and *expectations* about future relative net worth,  $\frac{W_{i,t+1}^I}{W_{i,t+1}^H}$ , and its co-movement with the Dollar, as captured by the covariance term. The fact that the absolute size of the Basis is proportional to  $\frac{W_{i,t}^H}{W_{i,t}^I}$  is intuitive. Indeed, this wealth ratio is an effective measure of intermediaries' bargaining power: When customers are rich and there are few intermediaries, each of them can extract larger rents. The link between the Basis and the *conditional* covariance of the *future* wealth ratio with the Dollar is also intuitive: When this covariance is negative, customers' wealth is low (or, equivalently, their marginal utilities are high) precisely in the states in which US Dollar is weak.<sup>38</sup> Naturally, customers would like to borrow in a currency that tends to depreciate relative to their domestic currency at times

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<sup>38</sup>Of course, this holds if the relationship between the wealth ratio and the dollar is linear. In this case, negative covariance means that the wealth ratio is high (respectively, low) when the dollar is weak (respectively, strong).

when they are distressed and their marginal utility is high.<sup>39</sup> Thus, a negative *expected* relationship between customers' wealth and the dollar makes borrowing in dollars attractive: Effectively, the Dollar is a hedge against distress states. Intermediaries cater to the demand for customers for dollar borrowing and find it optimal to extract markups for (synthetic) dollar lending. Interestingly enough, equation (16) implies that one could potentially use the observed dynamics of the Basis to infer about the (unobservable) wealth ratio.<sup>40</sup>

Suppose now that the US dollar co-moves negatively with the ratio  $W_{i,t}^I/W_{i,t}^H$ , and, hence, co-moves positively with  $W_{i,t}^H/W_{i,t}^I$ . Then, formula (16) implies that the absolute size of the Basis, being proportional to  $W_{i,t}^H/W_{i,t}^I$ , increases with  $\$_{i,t}$ , while the sign of the Basis, being equal the sign of the conditional covariance, is positive. We summarize these observations in the following proposition.

**Proposition 7** *Suppose that the US dollar co-moves negatively with the ratio  $W_{i,t}^I/W_{i,t}^H$ , so that US Dollar tends to appreciate when intermediary wealth drops relative to that of customers. Then,*

- *The basis is positive;*
- *The absolute size of the basis co-moves positively with the US dollar.*

The negative co-movement of the wealth ratio  $W_{i,t}^I/W_{i,t}^H$  with the dollar is consistent with the “safe haven” properties of the dollar: If country- $i$  intermediaries sell a lot of insurance against global crisis states, the ratio  $W_{i,t}^I/W_{i,t}^H$  will tend to drop in those states, while the dollar will appreciate in such states due to its safe haven status.<sup>41</sup>

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<sup>39</sup>Note that, of course, in equilibrium, the exchange rate depends on the precise relationship between the marginal utilities of customers in the two countries,  $i$  and US.

<sup>40</sup>See Hebert (2017) who shows how the cross-section of Basis can be used to extract information about intermediation capacity.

<sup>41</sup>Interestingly enough, the two predictions of Proposition 7 are consistent with recent empirical evidence that suggests that the Basis is indeed mostly positive and indeed co-moves with the US Dollar. See, Avdjiev et al. (2016).

### 3 Equilibrium

Equilibrium prices are pinned down by imposing market clearing for all goods. The total nominal expenditures for country  $k$  goods, measured in country  $k$  currency, are given by

$$\sum_i (C_{i,t}^H + C_{i,t}^I) \theta_{i,k} \mathcal{E}_{i,t}.$$

By the cash-in-advance constraint, total nominal expenditures equal money supply, and hence, using (6) and (7), we get that equilibrium state prices are pinned down by the equation system

$$\sum_{i=1}^N \Psi_{i,t} (C_{i,0}^H (M_{i,0,t}^H)^{-1} + C_{i,0}^I (M_{i,0,t}^I)^{-1}) \theta_{i,k} \mathcal{E}_{i,t} = \mathcal{M}_{k,t} \mathcal{E}_{k,t}, \quad k = 1, \dots, N,$$

with  $M_{i,0,t}^I$  given by (2). In general, the structure of equilibrium can be quite complex and depends in a non-trivial way on the distribution of preference parameters  $\theta_{i,k}$  across countries. To isolate the domestic demand/supply effects from those of global demand and supply, we will assume that consumption demand exhibits a single factor structure, so that

$$\theta_{i,k} = \bar{\theta}_k \beta_i + (1 - \beta_i) \delta_{i,k},$$

where  $\delta_{i,k}$  is the Kronecker delta, and  $\sum_{k=1}^N \bar{\theta}_k = 1$ . Here,  $1 - \beta_i$  measures the degree of consumption home bias in country  $i$ , while  $\bar{\theta}_k$  reflects the global demand for country  $k$  goods. In order to maintain analytical tractability, we will follow the approach of [Itskhoki and Mukhin \(2017\)](#) and study the behaviour of prices and exchange rates in the limit of a substantial home bias, corresponding to the case when  $\beta_i$  is small for all  $i$ .<sup>42</sup>

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<sup>42</sup>As [Itskhoki and Mukhin \(2017\)](#) argue, many countries exhibit significant home bias in consumption. We have also solved the opposite limit of vanishing home bias and most of our results qualitatively hold in this environment. We therefore expect that our results are robust to the degree of the home bias.

In our model, country  $i$  customers would only be willing to get exposure to foreign shocks if there would be possibilities for international trade. Indeed, in the strong home bias limit, as  $\beta_i \rightarrow 0$ , customers simply consume their endowment and there are no gains from trading.<sup>43</sup> Thus, intermediaries have nothing to charge markups for, and intermediation frictions do not have any impact on equilibrium prices. In particular, exchange rates are given by the ratios of discounted marginal utilities,<sup>44</sup>

$$\frac{\mathcal{E}_{j,t+1}^*}{\mathcal{E}_{j,t}^*} \equiv \frac{\mathcal{N}_{j,t+1}^{-1} \Psi_{i,t,t+1}}{\mathcal{N}_{\$,t+1}^{-1} \Psi_{\$,t,t+1}}. \quad (17)$$

With strong home bias, country  $i$  nominal consumption expenditures are approximately given by the money supply,  $\mathcal{M}_{i,t}$ , and a fraction  $\beta_i$  of these expenditures is spent on foreign goods. Thus,  $(1 - \bar{\theta}_i)\beta_i\mathcal{M}_{i,t}\mathcal{E}_{i,t}^*$  is the (approximate) total dollar value of country  $j$  expenditures on foreign goods, thus capturing the share of country  $i$  in international trade. In the sequel, we will refer to

$$Dollar_t \equiv - \sum_j \beta_j \mathcal{M}_{j,t} \mathcal{E}_{j,t}^*$$

the global, trade-weighted Dollar index. When US dollar appreciates relative to all other currencies,  $\mathcal{E}_{j,t}^*$  drop, and the dollar index  $Dollar_t$  rises in value.

We will also frequently use the domestic stock market prices in the strong home bias limit, which we will denote by  $S_{i,t}^*$ ,  $i = 1, \dots, N$ . For each  $i$ , the stock price  $S_{i,t}^*$  is proportional to money supply and is given by

$$S_{i,t}^* = \mathcal{M}_{i,t} D_{i,t}, \quad (18)$$

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<sup>43</sup>Recall that we assume that customers and intermediaries have identical discount factors, and therefore there are no gains from trading between these two groups within one country.

<sup>44</sup>See Appendix A.1 for a formal derivation.

with the discount factor  $D_{i,t}$  defined in (5). Indeed, due to the cash in advance constraint, the stock price is just the value of the claim on the future money supply. Because of log preferences, marginal utilities are inversely proportional to money supply, and hence stock prices move one-to-one with the time discount factor  $D_{i,t}$ . We then define the US Dollar price of the global trade-weighted stock market portfolio as follows:

$$\bar{S}_t^\$ \equiv \sum_j \beta_j S_{j,t}^\$, \quad (19)$$

where we have defined the US dollar prices of domestic stock market indices,

$$S_{j,t}^\$ = \mathcal{E}_{j,t}^* S_{j,t}^*.$$

Consistent with the notation in previous sections, we also define

$$Dollar_{t,t+1} \equiv \frac{Dollar_{t+1}}{Dollar_t}, \quad \bar{S}_{t,t+1}^\$ \equiv \bar{S}_{t+1}^\$/\bar{S}_t^\$, \quad S_{j,t,t+1}^\$ \equiv S_{j,t+1}^\$/S_{j,t}^\$.$$

Everywhere in the sequel, we will refer to

$$w_i^* \equiv W_{i,0}^I/W_{i,0}^H \quad (20)$$

as the country  $i$  intermediation capacity. This quantity captures the relative net worth of intermediaries relative to customers, and hence measures the ability of intermediaries to take on balance sheet risk when trading with customers. The following is true.

**Theorem 8** *Equilibrium domestic stock prices are given by*

$$S_{i,t} \approx S_{i,t}^* \left( 1 + \bar{\theta}_i \left( \frac{\bar{S}_t^\$}{S_{i,t}^\$} + \frac{Dollar_t}{\mathcal{M}_{i,t} \mathcal{E}_{i,t}^*} \right) \right), \quad (21)$$

while the country  $i$  D2D pricing kernel is given by

$$\begin{aligned}
M_{i,t,t+1}^I &\approx \mathcal{N}_{i,t+1}^{-1} \Psi_{i,t,t+1} \\
&\times \left( \frac{1}{2w_i^* + 1} \underbrace{(\lambda_{i,t} + \mu_{i,t}(S_{i,t,t+1}^*)^{-1})}_{\text{Shadow Cost of Intermediation}} - \bar{\theta}_i \frac{\text{Dollar}_t}{\mathcal{M}_{i,t} \mathcal{E}_{i,t}^*} \underbrace{\left( \frac{\text{Dollar}_{t,t+1}}{\mathcal{N}_{i,t+1} \mathcal{E}_{i,t,t+1}^*} - 1 \right)}_{\text{Dollar Factor}} \right) \\
&+ \underbrace{\frac{\bar{\theta}_i}{2w_i^* + 1} \frac{\bar{S}_t^\$}{S_{i,t}^\$} \left( \frac{\bar{S}_{t,t+1}^\$}{S_{i,t,t+1}^\$} - 1 \right)}_{\text{Global Trade-Weighted Stock Market Portfolio}}
\end{aligned}$$

while exchange rates changes are given by

$$\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}} = \frac{M_{i,t,t+1}^I}{M_{\$,t,t+1}^I}. \quad (22)$$

The intuition behind (21) is as follows: Since  $S_{i,t}$  is the nominal value of the claim on the money flow  $\mathcal{M}_{i,t}$ , it responds to trade exclusively through the pricing kernel<sup>45</sup>  $M_{i,t,t+1}^H$ . By market clearing,  $\bar{S}_t^\$ - \beta_i S_{i,t}^\$$  is precisely the dollar value of *future* consumption that domestic agents “give away” to foreign agents. Since  $M_{i,t,t+1}^H$  equals the agents’ marginal utility growth, a drop in consumption pushes marginal utilities and prices up.<sup>46</sup>

Theorem 8 shows that our model also generates a multi-factor model for the US dollar pricing kernel  $M_{\$,t,t+1}^I$ , with key risk factors given by (i) the shadow cost of intermediation; (ii) a trade-weighted dollar factor; and (iii) a trade weighted stock market factor. We now discuss the origins of these three factors in detail.

The shadow cost of intermediation factor, as given by

$$\lambda_{i,t} + \mu_{i,t}(S_{i,t,t+1}^*)^{-1},$$

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<sup>45</sup>Recall formula (4).

<sup>46</sup>The term  $\text{Dollar}_t$  appears because what matters for  $M_{i,t,t+1}^H$  is the difference between current and future consumption loss, and  $\text{Dollar}_t$  is the current consumption loss.



enters the pricing kernel because it impacts intermediary net worth. Importantly, this factor depends on the domestic stock market returns, and the exposure to these returns is determined by  $\mu_{i,t}$ . We derive an explicit expression for  $\mu_{i,t}$  and study its behaviour in the next section.

The trade-weighted dollar factor,  $Dollar_t$  appears in the pricing kernel because domestic consumption and domestic purchasing power depend on the (trade weighted) exchange rate index against the local currency. In particular, with the US dollar as the reference currency, the US dollar pricing kernel depends on the global dollar index. When this dollar index depreciates (i.e., dollar weakens), global demand for US goods goes up, while the US domestic consumption decreases.<sup>47</sup> As a result, insurance against dollar depreciation states becomes valuable for US customers, and the US dollar pricing kernel loads negatively on the dollar index.<sup>48</sup>

The last factor,

$$\frac{\bar{S}_{t,t+1}^{\$}}{S_{i,t,t+1}^{\$}},$$

originates purely from intermediation frictions and, as we explain above, captures shocks to domestic customers' net worth relative to the (trade weighted) global net worth. When the relative net worth,<sup>49</sup>  $\frac{\bar{S}_{i,t,t+1}^{\$}}{\bar{S}_{t,t+1}^{\$}}$ , is large, intermediaries are able to extract larger rents, become wealthier, and their marginal utilities drop, and hence the pricing kernel loads positively on the factor.

Interestingly enough, formula (22) implies that this intermediary net worth channel makes

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<sup>47</sup>See also [Gourinchas et al. \(2017\)](#) who show that the US dollar index is negatively related to global trade when US Dollar is an invoicing currency.

<sup>48</sup>The fact that Dollar is a priced factor is supported by the empirical evidence. [Verdelhan \(2017\)](#) finds that US dollar is an important risk factor explaining a significant fraction of the cross-section of currency returns. See also [Brusa et al. \(2014\)](#). [Hassan and Mano \(2017\)](#) link this result to the cross-section of currency risk premia.

<sup>49</sup>This relative net worth is simply the reciprocal of the factor  $\frac{\bar{S}_{t,t+1}^{\$}}{S_{i,t,t+1}^{\$}}$ .

exchange rates depend on expectations on future fundamentals and future interest rate differentials.<sup>50</sup> While this is well known (see, for example, [Froot and Ramadorai \(2005\)](#)) that exchange rates are naturally related to the value of future interest rate differentials, in the frictionless model this relationship is dominated by the instantaneous interest rate differential (the “pure” UIP). Theorem 8 shows that intermediation frictions do lead to a non-trivial relationship between exchange rates the present discounted value of future interest rate differentials.<sup>51</sup> However, this relationship is non-linear and may be time-varying, depending on the intermediation capacity. In particular, shocks to this capacity will generate endogenous flows between customers and intermediaries, changing the dynamics of exchange rates, in agreement with the findings of [Froot and Ramadorai \(2005\)](#). Importantly, all effects of intermediation frictions are monotone decreasing in the relative net worth  $w_i^*$  or intermediaries relative to that of customers (the intermediation capacity (20)). The higher this capacity, the smaller is the exposure of the pricing kernel (and, hence, the exchange rates) to the intermediation-driven shocks.

## 4 Intermediation and Exchange Rate Anomalies

In this section, we use our explicit expression for the equilibrium pricing kernel and exchange rates (Theorem 8) to investigate the role of intermediation frictions for various known anomalies in exchange rates and international capital markets. In this context, the impact of heterogeneity in country characteristics on risk properties of exchange rates is crucial.<sup>52</sup> Define the D2C risk neutral measure in the strong home bias limit,

$$d\tilde{P}_{i,t}^* \equiv \frac{M_{i,t,t+1}^{H,*}}{E_t[M_{i,t,t+1}^{H,*}]},$$

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<sup>50</sup>Indeed, by (5),  $S_{i,t}^* = \mathcal{M}_{i,t} D_{i,t}$  is nothing but the present value of future nominal rates.

<sup>51</sup>Note also that the presence of the third factor in exchange rates implies that exchange rates may forecast fundamentals in our model, consistent with the findings of [Engel and West \(2005\)](#).

<sup>52</sup>See, for example, [Lustig et al. \(2011\)](#) and [Hassan and Mano \(2017\)](#) who argue that heterogeneity in the cross-section is key to understanding exchange rate anomalies.

where we have defined the strong home bias domestic pricing kernel

$$M_{i,t,t+1}^{H,*} = M_{i,t,t+1}^{I,*} = \mathcal{N}_{i,t+1}^{-1} \Psi_{i,t,t+1}$$

and the normalization factor is just the nominal interest rate,

$$(E_t[M_{i,t,t+1}^{H,*}])^{-1} = e^{r_{i,t}^*}.$$

Everywhere in the sequel, we denote by  $\widetilde{E}$  and  $\widetilde{\text{Cov}}$  the expectation and the covariance under the  $\widetilde{P}_{i,t}^*$  measure. The following proposition provides an explicit expression for the shadow costs  $\mu_{i,t}$ ,  $\lambda_{i,t}$ .<sup>53</sup>

**Proposition 9** *The shadow costs  $\mu_{i,t}$ ,  $\lambda_{i,t}$  are given by*

$$\begin{aligned} \lambda_{i,t} &\approx 1 + \theta_i \left( \frac{\bar{S}_t^{\$}}{S_{i,t}^{\$}} - \frac{\widetilde{\text{Cov}}_t \left( S_{i,t+1}^* \frac{\bar{S}_{t+1}^{\$}}{S_{i,t+1}^{\$}}, 1/S_{i,t+1}^* \right)}{\widetilde{\text{Cov}}_t(S_{i,t+1}^*, 1/S_{i,t+1}^*)} \right) \\ \mu_{i,t} &\approx -\theta_i \frac{\widetilde{\text{Cov}}_t \left( \frac{\bar{S}_{t+1}^{\$}}{S_{i,t+1}^{\$}}, S_{i,t+1}^* \right)}{\widetilde{\text{Cov}}_t(S_{i,t+1}^*, 1/S_{i,t+1}^*)} \end{aligned} \quad (23)$$

The intuition behind (23) is as follows. The signs (and the size) of the shadow costs  $\lambda_{i,t}$ ,  $\mu_{i,t}$  depend on the ability of the stock market to serve as an efficient hedge against states with very high state prices. Naturally,  $\lambda_{i,t}$  is positive (at least for small international trade): intermediation markups make access to foreign claims costly. This forces customers to retain significant positive exposure to the domestic stock market (that is, the value of their endowment) leading to an *endogenous home bias*, simply because trading foreign securities entails (endogenous) transaction costs.<sup>54</sup>

<sup>53</sup>Note that, importantly, all the expressions in Proposition 9 involve only exogenous quantities.

<sup>54</sup>See, e.g., [Coeurdacier and Rey \(2013\)](#) for an overview of the literature on home bias.

By contrast, as we explain above, the sign of  $\mu_{i,t}$  depends on whether customers are net borrowers or net lenders in the domestic D2C market:<sup>55</sup> When customers buy bonds in the D2C market (i.e., they are long bonds and hence are net lenders),  $\mu_{i,t} > 0$ ; this is what we call a “risk-off” scenario. When customers sell bonds in the D2C market (i.e., they are short bonds and hence are net borrowers),  $\mu_{i,t} < 0$ ; this is what we call a “risk-on” scenario. The key implication of Proposition 9 is that international trade and capital flows trigger risk-on and risk-off scenarios depending on the customers’ expectations about the conditional covariance of their net worth (given by the domestic stock market,  $S_{i,t}^*$ ) with the relative contribution of their net worth to the global trade-weighted stock market portfolio,  $\frac{\bar{S}_t^\$}{S_{i,t}^\$}$ .

The intuition behind this result is as follows. As we explain above, shocks to the ratio  $\frac{\bar{S}_t^\$}{S_{i,t}^\$}$  reflect shocks to relative net worth: *Country  $i$  customers care about states in which their net worth drops relative to that of the rest of the world.* Indeed, in the presence of international trade, the domestic stock market is not anymore perfectly correlated with customers’ wealth. For this reason, customers need to contact intermediaries and buy state-contingent claims that partially offset the deviation of their net worth from the domestic stock market value. By formula (21), this deviation is driven by relative net worth  $\frac{\bar{S}_t^\$}{S_{i,t}^\$}$ .<sup>56</sup> When the ratio  $\frac{\bar{S}_t^\$}{S_{i,t}^\$}$  positively co-moves with  $S_{i,t+1}^*$ , country  $i$  customers perceive it as highly risky because states in which their net worth  $S_{i,t+1}^*$  drops coincide with the states in which their net worth drops relative to the rest of the world; as a result, customers enter a risk-off regime, reduce their exposure to the stock market, and create price pressure in the D2C market, making  $\mu_{i,t}$  positive. By contrast, when  $\frac{\bar{S}_t^\$}{S_{i,t}^\$}$  negatively co-moves with  $S_{i,t+1}^*$ , formula (21) implies that international trade decreases riskiness of the domestic portfolio: Indeed, in this case, a drop in the domestic net worth corresponds to an increase in customers net worth relative to rest of the world. This makes investing in the stock market highly attractive, and

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<sup>55</sup>See the discussion after formulas (11).

<sup>56</sup>Note that, while we are assuming that customers can freely trade only local stocks, giving them access to international stock markets will not make the market complete, for generic stochastic processes  $\Psi_{i,t}$ . Thus, customers will generally need to buy contracts providing non-linear exposures to international markets.

triggers a risk-on regime. Customers optimally increase their leverage by borrowing in the D2C market. This creates a price pressure in the D2C market, making  $\mu_{i,t}$  negative.

## 4.1 Global trade, wealth redistribution, and a dichotomy of countries

Absent intermediation frictions, changes in expectations about future monetary policy (e.g., through central bank forward guidance) do not have any effect on the exchange rates: With perfect markets, agents can efficiently share the aggregate risk, and nominal shocks are neutral. As we will show in this section, intermediation frictions make shocks to money supply non-neutral and *redistributive*: That is, shocks to money supply redistribute wealth across countries. This wealth redistribution operates through two channels: ex-ante expectations of shocks to domestic stock market value,  $S_{i,t}$ ; and ex-post impact of these shocks on the exchange rates,  $\mathcal{E}_{i,t}$ .<sup>57</sup>

The two channels interact through the balance sheets of intermediaries. Namely, expectations of future state-contingent monetary policy impacts customers' incentives to buy insurance and/or make levered bets with intermediaries. In turn, intermediaries offload their D2C positions in the D2D market. When the shock is realized next period, intermediaries make/receive payments on their cross-border positions, leading to an international redistribution of wealth.

This redistribution operates through the exchange rate channel because trade in the D2C market amounts to a redistribution of wealth between customers and intermediaries and hence keeps the total nominal country  $i$  wealth,  $W_{i,t+1}^H + W_{i,t+1}^I$ , unchanged in real terms: This wealth simply scales proportionally with money supply. At the same time, monetary shocks impact intermediaries' net worth and, hence, their marginal utilities. The latter, in turn, determine the exchange rate,  $\mathcal{E}_{i,t+1}$ . Thus, in our model, a *monetary shock*

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<sup>57</sup>We could, for example, interpret the former internal monetary shocks, and the latter as foreign exchange interventions.

that leads to exchange rate appreciation always makes the country richer in Dollar terms, pushing the US Dollar wealth,  $(W_{i,t+1}^H + W_{i,t+1}^I)\mathcal{E}_{i,t+1}$  up. This result is consistent with the findings of Lane and Shambaugh (2010) to show that the majority of countries are exposed to substantial valuation losses in the event of their home currency depreciation.

Since intermediaries have log utilities, exchange rates move inversely to intermediary wealth and, hence, exchange rate appreciation is always one-to-one with drops in intermediary net worth. By Lemma 3 and Proposition 4, the reciprocal of intermediary wealth change is proportional to the shadow cost of intermediation normalized by country  $i$  stock market returns,  $(W_{i,t+1}^I)^{-1} \sim (\lambda_{i,t} + \mu_{i,t}(S_{i,t,t+1}^{-1}))$ . The states in which this shadow cost is high (respectively, low) are the states in which intermediaries make payments to (respectively, receive payments from) customers. Since nominal stock prices move proportionally to money supply, we get that  $(W_{i,t+1}^I)^{-1} \sim \mu_{i,t}\mathcal{N}_{i,t+1}^{-1}$ . Hence, intermediary wealth responds positively to monetary shocks if and only if  $\mu_{i,t} < 0$ . By Proposition 9, this is equivalent to  $\widetilde{\text{Cov}}_t\left(\frac{\bar{S}_{t+1}^S}{S_{i,t+1}^S}, S_{i,t+1}^*\right) > 0$ . The goal of this section is to characterize the sign of this covariance and link it to state-contingent monetary policy.

In the sequel, we will always make the following technical assumption.<sup>58</sup>

**Assumption 3** *There exists a Markov process  $\omega_t \in \mathbb{R}$ ,  $t \geq 0$  with mean  $\bar{\omega}$ , and  $N$  sequences  $\varepsilon_{i,t}$  or day random shocks that are independent across  $t$  and  $i$ , such that*

$$\log \Psi_{i,t,t+1} = \delta_i^\Psi \omega_{t+1} + \varepsilon_{i,t+1}$$

for some  $\delta_i^\Psi > 0$ ,  $i = 1, \dots, N$ .

Assumption 3 simply imposes a single factor structure in international time discount rates  $-\log \Psi_{i,t,t+1}$ . The coefficients  $\delta_i^\Psi$  are the sensitivities of domestic discount rates to the global factor  $\omega_{t+1}$ , while  $\varepsilon_{i,t+1}$  are country-specific, idiosyncratic shocks. It is straightforward to

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<sup>58</sup>This assumption can be relaxed at the cost of imposing additional technical conditions on the transition probabilities of the underlying processes.

show<sup>59</sup> that equilibrium stock prices inherit this factor structure. Our goal is to understand the link between different countries' demand for insurance and their exposure to global trade-weighted stock market portfolio. By the definition (19) of the global trade-weighted stock market portfolio, we have

$$\sum_j \beta_j \frac{S_{j,t}^\$}{\bar{S}_t^\$} = 1,$$

and therefore,

$$\sum_j \beta_j \frac{\partial}{\partial \omega_t} \frac{S_{j,t}^\$}{\bar{S}_t^\$} = 0. \quad (24)$$

By (24), the average (trade-weighted) sensitivity of countries relative stock prices (in US Dollars),  $\frac{S_{j,t}^\$}{\bar{S}_t^\$}$ , is zero, and, hence, we can classify countries according to their exposure to global shocks, relative to that of the global trade-weighted stock market portfolio.

**Definition 10** *A country  $i$  is strongly (respectively, weakly) exposed to global shocks if  $\frac{\partial}{\partial \omega_t} \frac{S_{j,t}^\$}{\bar{S}_t^\$} |_{\bar{\omega}} > 0$  (respectively,  $< 0$ ).*

It is straightforward from (24) that the country with the highest (respectively, lowest) sensitivity of stock prices to  $\omega$  is always strongly (respectively, weakly) exposed to global shocks. However, for countries with intermediate levels of sensitivity to  $\omega$  the behaviour is more subtle and depends on each country's wealth share in the global trade-weighted stock market portfolio.

As we will now show, countries that are weakly or strongly exposed to global shocks exhibit opposite behaviour in their demand for insurance against these shocks. In order to derive this dichotomy of behavior, we need to specify how monetary policies are conducted across countries.

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<sup>59</sup>See Lemma 19 in the Appendix.

In our model, the dynamics of asset prices and exchange rates are influenced by two primitive risk factors: (i) shocks to discount rates, as captured by the stochastic nature of  $\Psi_{i,t}$ ; and (ii) shocks to monetary policy, as captured by the stochastic nature growth in the money supply,  $\mathcal{N}_{i,t}$ . The former play the role of sentiment/demand shocks; the latter are supposed to respond to these demand shocks, stabilizing the economy and protecting it from “overheating” and “depressions”. By (18), we have  $S_{i,t,t+1}^* = S_{i,t+1}^*/S_{i,t}^* = \mathcal{N}_{i,t+1} D_{i,t,t+1}$ . That is, the stock market return is the product of discount factor shocks and monetary policy shocks. Thus, a policy that sets the money growth,  $\mathcal{N}_{i,t+1}$ , to be monotone decreasing in  $\omega_t$  naturally dampens volatility of the stock prices.<sup>60</sup> Following Hassan et al. (2016), we will use the term “stabilization policy” to describe such a state contingent monetary policy.<sup>61</sup> Define

$$\delta_i \equiv \frac{\partial}{\partial \omega_t} \log D_{i,t} |_{\omega_t = \bar{\omega}} > 0.$$

Everywhere in the sequel, we will make the following simplifying assumption.

**Assumption 4** *Domestic stabilization policies follow*

$$\log \mathcal{N}_{i,t+1} = -\alpha_i^{\mathcal{N}} \delta_i \omega_t + \varepsilon_{i,t+1}^{\mathcal{N}}, \quad (25)$$

where  $\alpha_i^{\mathcal{N}}$  is the sensitivity of the country- $i$  policy to global shocks, and  $\varepsilon_{i,t+1}^{\mathcal{N}}$  are idiosyncratic

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<sup>60</sup>Note that, importantly, the fact that  $\mathcal{N}_{i,t+1}$  is monotone decreasing in  $D_{i,t,t+1}$  does not mean that the monetary policy directly reacts to the stock market (even though this does seem to be the case; see, Cieslak and Vissing-Jorgensen (2017)). What it actually means is that the monetary policy reacts to the economic conditions (as captured by the shocks to  $\Psi_{i,t}$ ) which are also reflected in  $D_{i,t,t+1}$ . Since both  $\mathcal{N}_{i,t+1}$  and  $D_{i,t,t+1}$  react to the same macroeconomic shocks, a stabilization policy will always look as if it is “leaning against the wind”, stabilizing the stock market. See, for example, Law et al. (2017) for evidence that US monetary policy does react strongly to the macroeconomic situation.

<sup>61</sup>Reinhart and Rogoff (2004) show that 88% of countries (representing 47% of world GDP) stabilize their currency relative to some target country. As Hassan et al. (2016) argue, adjusting money supply in response to shocks can be interpreted as direct (nominal) currency interventions.



monetary shocks that are independent across  $t$  and  $i$ . We say that the stabilization policy is mild (respectively, strong) if  $\alpha_i^N < 1$  (respectively,  $\alpha_i^N > 1$ ).

In the frictionless economy, exchange rates  $\mathcal{E}_{i,t+1}$  are inversely proportional to money supply, so that  $\frac{\partial \log \mathcal{E}_{i,t+1}}{\partial \log(\mathcal{N}_{i,t+1}^{-1})} = 1$ . We will say that exchange rate “overshoots” in response to monetary shocks if  $\frac{\partial \log \mathcal{E}_{i,t+1}}{\partial \log(\mathcal{N}_{i,t+1}^{-1})} > 1$ . As we explain above, such an effect in our model originates in (endogenous) shocks to intermediary balance sheets and is proportional to  $\mu_{i,t} \mathcal{N}_{i,t+1}^{-1}$ . Thus, exchange rates overshoot if and only if  $\mu_{i,t} > 0$ . For small variance of shocks,  $\mu_{i,t} \sim \widetilde{\text{Cov}}_t \left( \frac{\bar{S}_{t+1}^S}{S_{i,t+1}^S}, S_{i,t+1}^* \right)$  is positive if and only if  $\frac{\bar{S}_{t+1}^S}{S_{i,t+1}^S}$  and  $S_{i,t+1}^*$  have the opposite signs of exposure to global shocks. Since the sign of the exposure of  $S_{i,t+1}^* = \mathcal{N}_{i,t+1} D_{i,t,t+1}$  is positive if and only if the stabilization policy is mild, we arrive at the following result.

**Proposition 11** *Suppose that the variance of all shocks is small. The following is true if and only if either (a) the stabilization policy in country  $i$  is mild and it is weakly exposed to global shocks; or (b) the stabilization policy in country  $i$  is strong and it is strongly exposed to global shocks:*

- (1) the exchange rate  $\mathcal{E}_{i,t}$  “overshoots” in response to country  $i$  monetary shocks;
- (2) the total country  $i$  US dollar wealth,  $(W_{i,t+1}^H + W_{i,t+1}^I) \mathcal{E}_{i,t+1}$ , decreases in country  $i$  monetary shocks.<sup>62</sup>

The strength of these effects is decreasing in country  $i$  intermediation capacity,  $w_i^*$ .

The intuition behind Proposition 11 is as follows. The dichotomy of Definition 10 directly transmits into an analogous dichotomy for the insurance demand in the D2C market and the opposing reactions of countries to monetary policy: By Proposition 9, if money supply

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<sup>62</sup>We interpret a positive shock to money supply,  $\mathcal{N}_{i,t+1} > 1$ , as monetary policy easing: Since the frictionless exchange rates scale inversely with money supply, this is consistent with empirical evidence (see Ferrari et al. (2017)) indicating that monetary policy easing shocks indeed always lead to an immediate contemporaneous depreciation of exchange rates. Thus, item (2) means that an easing shock (money expansion) leads to a depreciation of both exchange rate and domestic wealth.

“overshoots”, the sign of customers’ demand for leverage flips, pushing customers into a “risk-on” regime. Intermediaries respond by charging high markups for the access to the “upside” (e.g., through out-of-the-money call options) corresponding to states with high realizations of  $D_{i,t+1}$ . In this case,  $S_{i,t+1}^*/S_{i,t}^* = \mathcal{N}_{i,t+1} D_{i,t,t+1}$  is monotone *decreasing* in  $D_{i,t,t+1}$  : That is, the monetary policy is so strong that it reverses the sign of the reaction of the stock market to fundamental shocks.<sup>63</sup> Importantly, the strength of this effects decreases in intermediation capacity: When the latter is large, intermediaries are better able to absorb the shocks, and overshooting disappears.

While exchange rate depreciation has a positive impact on net exports, it damages the external balance sheet through a negative valuation effect; Proposition 11 derives necessary and sufficient conditions under which the valuation channel dominates the exports channel. Investigating whether the conditions of Proposition 11 indeed hold empirically is an interesting direction for future research.

It is instructive to discuss the relationship between the mechanisms underlying the result of Proposition 11 and the classical [Dornbusch \(1976\)](#) overshooting model. According to [Dornbusch \(1976\)](#), nominal price stickiness of tradable goods implies that, after a monetary shock, the short run equilibrium will first be achieved through shifts in financial market prices, exchange rates overreact to the shock in the short run. In contrast to [Dornbusch \(1976\)](#), goods prices are fully flexible in our model. Yet, exchange rates may over- or under-react to monetary shocks due to an endogenous “markup stickiness” that arises because the contracts between customers and intermediaries are signed ex-ante, before the monetary shock is realized and hence are naturally “sticky”. An unanticipated shock hits intermediaries’ balance sheets and their risk bearing capacity, leading to a repricing in the foreign exchange market.<sup>64</sup> The reaction depends crucially on the exposure of the country to

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<sup>63</sup>As [Law et al. \(2017\)](#) show, such scenarios are not uncommon and have occurred multiple times in the history of the US monetary policy.

<sup>64</sup>The fact that, in the presence of a strong stabilization policy, exchange rates may under-react to shocks implies that such strong policies may generate “momentum”-type effects that may also spillover onto other

global shocks because this exposure impacts the insurance demand of country  $i$  customers ex-ante. Note also that overshooting leads to excess sensitivity of exchange rates to monetary shocks and, hence, may contribute to their excess volatility. By Proposition 11, the strength of this effect decreases with country  $i$  intermediation capacity: As the latter increases, both overshooting and excess volatility vanish.

## 4.2 Safe haven currencies and monetary policy uncertainty

There is a large literature investigating differences in the stochastic properties of exchange rates across countries and, in particular, the tendency of some currencies to appreciate in bad times. Many explanations for this behaviour have been proposed, including differences in intermediaries' risk bearing capacity (Gourinchas et al. (2010) and Maggiori (2013)), country size (Martin (2012), Hassan (2013)), factor endowments (Ready et al. (2017), Powers (2015)), sensitivity to disaster risk (Farhi and Gabaix (2016)), trade centrality (Richmond (2015)), and exposure to long run risk (Colacito et al. (2017)). In this section, we propose a new driver of stochastic properties of exchange rates: monetary policy uncertainty. Our goal is to characterize the so-called "safe haven currencies" that appreciate in times of global market turmoil.

**Definition 12** *We say that currency  $i$  is safe haven relative to currency  $j$  if, conditional on time- $t$  information and absent monetary shocks, the relative exchange rate changes,  $\mathcal{E}_{i,t,t+1}/\mathcal{E}_{j,t,t+1}$  co-move negatively with the global stock market.*<sup>65</sup>

Since our focus is on monetary policy uncertainty, throughout this section we assume that countries  $i$  and  $\$$  only differ in the distribution of monetary policy shocks,  $\varepsilon_{i,t+1}^N$  and

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asset classes such as stocks. As we explain above, stabilization policies in our model can be interpreted both as state-contingent (domestic) monetary policy as well as direct foreign exchange interventions. Thus, Proposition 11 suggests that there may be a link between the strength of currency interventions and currency momentum, consistent with the findings of Menkhoff et al. (2012).

<sup>65</sup>See Assumption 3.

$\varepsilon_{\$,t+1}^{\mathcal{N}}$ . Thus, we assume that the countries have identical discount factors,  $D_t^* \equiv D_{i,t} = D_{\$,t}$ , identical intermediation capacities,

$$w^* = W_{\$,0}^I / W_{\$,0}^H,$$

and stabilization policies of identical strength,  $\alpha^{\mathcal{N}} = \alpha_i^{\mathcal{N}} = \alpha_{\$}^{\mathcal{N}}$ . In this case, Theorem 8 implies that the appreciation rate of the foreign currency is equal to

$$\mathcal{E}_{i,t,t+1} = \frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}} \approx \frac{\mathcal{N}_{i,t+1}^{-1}}{\mathcal{N}_{\$,t+1}^{-1}} \left( 1 + \frac{\bar{\theta}}{2w^* + 1} \left( \lambda_{i,t} - \lambda_{\$,t} + (D_{t,t+1}^*)^{-1} (\mu_{i,t} \mathcal{N}_{i,t+1}^{-1} - \mu_{\$,t} \mathcal{N}_{\$,t+1}^{-1}) \right) \right).$$

Thus, absent monetary shocks, we have

$$\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}} \approx \left( 1 + \frac{\bar{\theta}}{2w^* + 1} \left( \lambda_{i,t} - \lambda_{\$,t} + (D_{t,t+1}^*)^{-1} e^{\alpha^{\mathcal{N}} \delta \omega} (\mu_{i,t} - \mu_{\$,t}) \right) \right). \quad (26)$$

Hence, assuming that the stabilization policies are mild ( $\alpha^{\mathcal{N}} < 1$ ), we get that the US Dollar is a safe haven relative to currency  $i$  if and only if  $\mu_{i,t} < \mu_{\$,t}$ : Indeed, in this case formula (26) implies that  $\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}}$  is monotone increasing in  $\omega$ , implying that US Dollar value is decreasing in  $\omega$ . We will say that country  $j$  has less policy uncertainty than country  $i$  if

$$\text{Var}_t[e^{\varepsilon_{j,t+1}^{\mathcal{N}}}] < \text{Var}_t[e^{\varepsilon_{i,t+1}^{\mathcal{N}}}].$$

The following result follows then from Proposition 9 by direct calculation.

**Proposition 13** *Suppose that (25) holds and that the countries only differ in the degrees of policy uncertainty, and that the countries  $i, \$$  are weakly exposed to global shocks. Then, US dollar is a safe haven currency relative to country  $i$  if and only if there is less uncertainty in US policy.*

In the frictionless economy, monetary policy uncertainty is irrelevant in the setup of

Proposition 13: Absent differences in time discount factors  $\Psi_i, \Psi_{\$}$ , exchange rates (17) satisfy  $\frac{\mathcal{E}_{i,t+1}}{\mathcal{E}_{i,t}} = \frac{\mathcal{N}_{\$,t+1}}{\mathcal{N}_{i,t+1}}$ . That is, when US money supply grows (relative to country  $i$ ), US Dollar depreciates (relative to country  $i$  currency). Hence, exchange rate movements are purely driven by monetary shocks and do not correlate at all with fundamental shocks  $\omega$ . Proposition 13 shows that intermediation frictions break this neutrality result; in fact, monetary policy uncertainty is bad for economic stability when countries do not profit from global expansions (item (1)).<sup>66</sup>

The mechanism underlying the result of Proposition 13 relies on the expectations channel. Customers anticipating a higher policy uncertainty contact intermediaries to buy insurance against future shocks. Intermediaries charge markups for providing this insurance, in turn limiting customers' ability to efficiently allocate consumption across future states and buy insurance against global stock market crashes. Proposition 13 implies that these distortions increase with the amount of monetary policy uncertainty.

Put differently, customers in *countries with greater monetary policy uncertainty are less able to insure against global shocks, and their consumption is more sensitive to these shocks*.<sup>67</sup> When an adverse shock hits global markets, intermediaries in all countries suffer and see their balance sheets shrink, while marginal utilities go up. However, intermediaries in countries with greater policy uncertainty suffer *less* than those in the US because they have sold less insurance against those global crisis states. This means that the exchange rate – given by the ratio of intermediaries' marginal utilities – depreciates relative to the safer US dollar.

Thus, as in Gourinchas et al. (2010) and Maggiori (2013), US dollar may become endogenously special in our model because US intermediaries act as global insurance providers. However, the underlying mechanism is different; namely, while Gourinchas et al. (2010) and

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<sup>66</sup>There is ample empirical evidence suggesting that policy uncertainty is important for the transmission of monetary shocks. See, for example, Arbatli et al. (2017) for recent evidence for Japan.

<sup>67</sup>The result of Proposition 13 is also broadly consistent with international evidence. For example, in emerging markets monetary policy tends to be less predictable, while the interest rates are high. Of course, there are many other factors distinguishing emerging market economies, such as higher inflation and greater marginal productivity of capital.

Maggiore (2013) assume that US intermediaries are special because they have a higher risk bearing capacity, in our model intermediaries in both countries are identical ex-ante, but behave differently ex-post because of different expectations about future domestic monetary policy. Note finally that the result of Proposition 13 depends on the assumption that US is weakly exposed to global shocks. The latter is equivalent to a *negative* conditional covariance between  $\frac{S_{\$,t,t+1}}{\bar{S}_{\$,t,t+1}^{\$}}$  and  $\bar{S}_{t,t+1}^{\$}$ . While formally testing this is outside of the scope of this paper, we have done some preliminary analysis by computing the correlation of the US market returns net of the world market returns, with the world market returns, over the period 1990-2017. This correlation is -25%, consistent with the assumption of Proposition 13.

### 4.3 Violations of covered interest parity

Large and systematic violations of the covered interest parity (CIP) condition represent one of the most puzzling anomalies in the post 2008 global financial crisis period.<sup>68</sup> Namely, the basis (defined as the difference between the swap-implied and the spot dollar rates; see formula (16)) is systematically positive for a large set of currencies.<sup>69</sup> In our model, *CIP deviations arise because of the price pressure created by the demand of customers in country  $i$  in the D2C market*. While in our stylized model this demand is formulated in terms of Arrow securities, one could envision two possible real-world interpretations. First, customers would like to borrow in US Dollars, but cannot do so directly and hence need to contact intermediaries. Alternatively, customers willing to invest into US Dollar denominated assets need to hedge the underlying FX risk, creating a US Dollar demand pressure in the swap market, which is in turn exploited by intermediaries.

As we show in Section 2.4 above (see Proposition 7), the basis for country  $i$  is positive if and only if the US dollar co-moves negatively with the ratio  $W_{i,t}^I/W_{i,t}^H$ . In turn, ignoring differences in discount factors  $\Psi_{i,t}$  and  $\Psi_{\$,t}$ , the US dollar is driven by the wealth ratio of

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<sup>68</sup>See, for example, Du et al. (2016), Avdjiev et al. (2016), Borio et al. (2016), and Rime et al. (2017).

<sup>69</sup>See formula (14) for a formal definition of basis.

US and country- $i$  intermediaries:

$$\mathcal{E}_{i,t} \sim \frac{W_{\$,t}^I}{W_{i,t}^I}.$$

Thus, formula (16) implies that  $Basis_{i,t}$  is positive if and only if  $\frac{W_{i,t}^I}{W_{\$,t}^I}$  co-moves negatively with  $\frac{W_{i,t}^I}{W_{\$,t}^I}$ . In this section, we will use Theorem 8 and Proposition 9 to study the joint dynamics of  $\frac{W_{i,t}^I}{W_{\$,t}^I}$  and  $\frac{W_{i,t}^I}{W_{i,t}^H}$  and the implied CIP deviations.

As above, in this section we assume in this section that country  $i$  follows the stabilization policy (25):

$$\log \mathcal{N}_{i,t+1} = -\alpha_i^{\mathcal{N}} \delta_i \omega_t + \varepsilon_{i,t+1}^{\mathcal{N}}.$$

Recall also that we use  $\delta_i = \frac{\partial}{\partial \omega_t} \log D_{i,t} |_{\omega_t = \bar{\omega}} > 0$  to denote the exposure of country  $i$  stock market to global shocks, while we will use

$$\delta_i^S \equiv \frac{\partial}{\partial \omega_{t+1}} \frac{S_{i,t,t+1}^{\$}}{\bar{S}_{t,t+1}^{\$}} |_{\omega_{t+1} = \bar{\omega}}$$

to denote country  $i$  sensitivity to shocks, relative to that of the global trade-weighted stock market portfolio  $\bar{S}_t^{\$}$ . By (24), we always have

$$\sum_i \beta_i \delta_i^S = 0,$$

and hence there is always a dichotomy between countries with positive and negative  $\delta_i^S$  (see Definition 10). Similarly, we will define

$$\delta_i^{\mathcal{N}} \equiv \frac{\partial}{\partial \omega_{t+1}} S_{i,t,t+1}^{*\mathcal{N}} |_{\omega_{t+1} = \bar{\omega}}.$$

While  $\delta_i^S$  is a real object that is independent of the monetary policy,  $S_{i,t,t+1}^* = \mathcal{N}_{i,t+1} D_{i,t,t+1}$  and hence  $\delta_i^N = (1 - \alpha_i^N) \delta_i$  depends on the state-contingent stabilization policy (25).

Define an auxiliary object<sup>70</sup>

$$Q_i \equiv \frac{0.5 \bar{\theta}_i (w_i^* + 2)}{w_i^* + 0.5},$$

The following Proposition shows how the total wealth of a country  $i$  (as captured by its total stock market capitalization,  $S_{i,t}^S$ ) and the two sensitivities,  $\delta_i^N$  and  $\delta_i^S$ , interact to determine the size and the magnitude of the basis.

**Proposition 14** *Suppose that the variances of all shocks are small. Then, we have<sup>71</sup>*

$$Basis_{i,t}^S \approx 0.5 \frac{\bar{S}_t^S}{S_{\$,t}^S} \left( \delta_i^S Q_i \frac{S_{\$,t}^S}{S_{i,t}^S} (\delta_i^S - \delta_i^N) - \delta_{\$}^S Q_{\$} (\delta_{\$}^S - \delta_{\$}^N) \right) E_t[(\Delta \omega_{t+1})^2].$$

If  $\delta_i^S (\delta_i^S - \delta_i^N) > 0$ , a positive Basis emerges if countries differ in only one of the following:

- (1) country  $i$  has smaller intermediation capacity than the US;
- (2) US has a higher market capitalization than country  $i$ ;

The key implication of our model is that this price pressure is determined by risk properties of Dollar.<sup>72</sup> In case (1) of Proposition 14, US dollar is attractive because high intermediation capacity makes exchange rates less sensitive to shocks (see Proposition 11). A good example for case (2) is an emerging market economy with a stock market that is highly sensitive to global demand shocks (for instance because its exports of commodities are not diversified enough). Customers in such a country depend crucially on the global demand for their goods; When the US market is a key component of this demand, customers

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<sup>70</sup>Clearly,  $Q_i$  is monotone decreasing in  $w_i^*$ , the intermediation capacity of country  $i$ ; see (20).

<sup>71</sup>Note that, by definition,  $S_{\$,t}^S = S_{\$,t}$  because stock prices  $S_{i,t}$  are in the domestic currency.

<sup>72</sup>Since the Basis is a form of risk premium, its size is proportional to the conditional variance of the shock,  $E_t[(\Delta \omega_{t+1})^2]$ .



find it optimal to borrow in US dollars. The demand pressure for US dollars will lead intermediaries in the FX swap market to adjust their quotes to avert flow imbalances and to extract markups as a compensation for lending their balance sheet.

#### 4.4 The global monetary spillover matrix

In the frictionless model, monetary policy is neutral: Money is just a numeraire and only impacts domestic nominal prices; all international effects are fully absorbed by exchange rates, keeping the real asset values unchanged. By contrast, with intermediation frictions, exchange rates may under- or over-shoot (see Proposition 11) because monetary shocks impact intermediary net worth. When financial markets are inter-connected, intermediaries endogenously create a complex network of mutual claims on each other, denominated in different currencies. Through this network, monetary shocks in one country affect intermediaries in all other countries, leading to an endogenous matrix of international monetary policy spillovers. See, for example, Bruno and Shin (2014), Avdjiev et al. (2016), and Shin (2017). The following result characterizes the equilibrium spillover matrix.

**Proposition 15** *Ceteris paribus, the sensitivity of (a) country  $i$  nominal bond prices; and (b) the net worth of country  $i$  customers,  $W_{i,t}^H$ , to a US monetary shocks is monotone increasing in*

- (1) *country  $i$  intermediation capacity,  $w_i^*$ .*
- (2) *country  $i$  stock market capitalization,  $S_{i,t}^S$ .*

The intuition behind the results of Proposition 15 is straightforward: A country with larger intermediation capacity is better integrated in the global economy and, as a result, responds more to foreign monetary shocks. Thus, while clearly improving risk sharing, larger intermediation capacity amplifies the transmission of international shocks. These results are

consistent with the recent findings of [Kearns et al. \(2018\)](#), who identify greater spillovers for advanced economies recipient countries.

## 5 Conclusions

We introduce an imperfectly competitive intermediation sector into a standard, international monetary model a-la [Lucas \(1982\)](#). We show that one simple friction, whereby intermediaries exploit their market power and charge endogenous markups for providing customers access to foreign securities, is able to generate a rich behaviour of risk premia and exchange rates.

The simple intermediation friction helps account for some of the major anomalies in foreign exchange and international capital markets, such as the safe haven properties of exchange rates, and the breakdown of covered interest parity (CIP). Our model shows explicitly how the nature of stabilization policies (including foreign exchange interventions) generate cross-sectional heterogeneity in a currency's risk profile and influences international risk sharing: customers in a particular country shift between risk-on or risk-off behaviour depending on how they perceive the evolution of their future wealth relative to the contribution of their own net worth to global wealth (in trade-weighted terms). Finally, our model endogenously generates a multi-factor pricing kernel, with the US dollar being an important priced risk factor.

# A Additional Material

The Appendix contains additional material:

- Section [A.1](#) reviews the frictionless model.
- Section [A.2](#) discusses exchange rate disconnect.
- Section [A.3](#) discusses crash risk.
- Section [B](#) provides an alternative foundation for downward sloping demand in the D2C market, based on portfolio constraints.
- Section [C](#) contains proofs of all results.

## A.1 Frictionless Economy

In this section, we solve for the equilibrium in the special case when there are no intermediation frictions and customers can freely trade with each other. This analysis serves as an important benchmark for the analysis in the main text. In this case, market completeness implies that all local nominal pricing kernels are linked through the state-by-state relationship with the US dollar pricing kernel:

$$M_{i,0,t}^H = M_{\$,0,t}^H \mathcal{E}_{i,t} / \mathcal{E}_{i,0}.$$

Furthermore, local nominal pricing kernels are determined by the cash-in-advance constraint,

$$\sum_i C_{i,0}^H \Psi_{i,t} (M_{i,0,t}^H)^{-1} \theta_{i,k} \mathcal{E}_{i,t} = \mathcal{M}_{k,t} \mathcal{E}_{k,t}.$$

so that

$$M_{k,0,t}^H = (\mathcal{M}_{k,t})^{-1} \Theta_{k,t}, \tag{A.27}$$

while the exchange rates are then given by

$$\mathcal{E}_{k,t} = \frac{M_{k,0,t}^H}{M_{\$,0,t}^H} = \frac{\mathcal{M}_{\$,t}}{\mathcal{M}_{k,t}} \frac{\Theta_{k,t}}{\Theta_{\$,t}}, \quad (\text{A.28})$$

where we have defined

$$\Theta_{k,t} \equiv \sum_i \mathcal{E}_{i,0} C_{i,0} \Psi_{i,t} \theta_{i,k}, \quad k = 1, \dots, N$$

to be the international wealth-weighted discount factor for goods of country  $k$ .

Money is super-neutral<sup>73</sup> in the frictionless economy, and both goods prices and nominal stock prices are proportional to money supply. The fact the money super-neutrality holds in frictionless cash-in-advance economies is well known: Money simply serves as a numraire, and has no impact on real asset prices. Similar arguments concern the other phenomena: Exchange rates exhibit a trivial behaviour and simply reflect preferences for local goods, with the parameters  $\theta_{k,t}$  being the primitive drivers of exchange rates dynamics. Furthermore, exchange rates perfectly perform their role of shock absorbers: Flexible exchange rates and capital flows guarantee monetary policy independence, as in Obstfeld and Taylor (2004) and in complete agreement with the Mundellian trilemma.

These simplistic features of the benchmark frictionless model are very useful for the analysis of the model with intermediation frictions: Indeed, they immediately imply that any interesting dynamic properties of prices and exchange rates are due solely to the intermediation frictions. We summarize these observations in the following proposition.

**Proposition 16 (Frictionless economy)** *The following is true in a frictionless economy in which customers can freely trade all securities with each other:*

- (1) *Money is super-neutral: nominal pricing kernels (A.27) are inversely proportional to*

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<sup>73</sup>Money is said to be super-neutral when neither the current money supply nor the expectations about the future monetary policy have any impact on real (inflation-adjusted) asset prices.

money supply, while nominal prices of real goods as well as stock prices are proportional to money supply:

$$P_{i,k,t} = \frac{\mathcal{M}_{i,t}}{X_{k,t}} \frac{\Theta_{k,t}}{\Theta_{i,t}^H}$$

$$S_{i,t} = \mathcal{M}_{i,t} E_t \left[ \sum_{\tau=t}^T \frac{\Theta_{i,\tau}^H}{\Theta_{i,t}^H} \right]$$

In particular, domestic inflation, stock prices, and the domestic pricing kernel are independent of foreign monetary policy shocks.

(3) Exchange rates are given by (A.28).

The following corollary summarizes basic properties of exchange rates in the frictionless economy.

**Corollary 17** *In a frictionless economy,*

- *The exchange rate  $\mathcal{E}_{i,t}$  always scales inversely with relative money supply. In particular, if country  $i$  expands the monetary base more than the US, then its currency always depreciates relative to US dollar.*
- *Expectations about future monetary policy (forward guidance) have no impact on exchange rates: They only depend on current money supply.*
- *Monetary shocks outside of US and country  $i$  have no impact on  $\mathcal{E}_{i,t}$ .*

## A.2 The disconnect of exchange rates and consumption

As in [Gabaix and Maggiori \(2015\)](#), in our model intermediaries are marginal investors in the international financial markets, and hence exchange rates are determined by their marginal

utilities which can be quite different from those of households. Specifically, we have

$$M_{i,t,t+1}^I = \Psi_{i,t,t+1}(C_{i,t+1}^I/C_{i,t}^I)^{-1} \neq \Psi_{i,t,t+1}(C_{i,t+1}^H/C_{i,t}^H)^{-1},$$

and hence

$$\mathcal{E}_{i,t+1}/\mathcal{E}_{i,t} = \frac{M_{i,t,t+1}^I}{M_{\$,t,t+1}^I} = \frac{\Psi_{i,t,t+1}(C_{i,t+1}^I/C_{i,t}^I)^{-1}}{\Psi_{\$,t,t+1}(C_{\$,t+1}^I/C_{\$,t}^I)^{-1}} \neq \frac{\Psi_{i,t,t+1}(C_{i,t+1}^H/C_{i,t}^H)^{-1}}{\Psi_{\$,t,t+1}(C_{\$,t+1}^H/C_{\$,t}^H)^{-1}}.$$

Thus, our model is naturally able to generate deviations from the one-to-one relationship between exchange rates and consumption, known as the [Backus and Smith \(1993\)](#) puzzle.

Consider a simplified setup in which two countries,  $i$  and  $j$ , have identical discount factors  $\Psi_{i,t}^H = \Psi_{j,t}^H$  and hence their only differences stem from monetary policies. By the cash in advance constraint, aggregate nominal consumption  $C_{i,t} = C_{i,t}^I + C_{i,t}^H$  coincides with the money supply, and hence  $C_{i,t+1}/C_{i,t} = \mathcal{N}_{i,t+1}$ . As a result, in the frictionless model, the correlation of exchange rates with relative consumption growth equals one, in stark contrast with the empirical evidence where this correlation is almost always negative (see, e.g., [Backus and Kehoe \(1992\)](#)). Here, we note that our model is also able to generate zero or negative correlation. For example, if the countries have identical monetary policies, so that  $\mathcal{N}_{i,t+1} = \mathcal{N}_{\$,t+1}$ , then  $(C_{i,t+1}/C_{i,t})/(C_{\$,t+1}/C_{\$,t}) = 1$  and hence its correlation with exchange rates is zero. At the same time, if intermediaries in the two countries are different, then exchange rates will exhibit non-trivial dynamics, unrelated to relative consumption.

### A.3 Crash risk

As we explained above, the state-contingent intermediation markups (12) represent the cost of insurance in the D2C market segment: when this cost is high, customers reduce their consumption in those states, driving down the value of the local currency. This fact has an important link with the empirical regularity known as the negative currency skew: That is,

the fact that, for many currencies, implied volatilities for out of the money put options tend to be higher than those for out of the money calls (see, e.g., [Farhi et al. \(2015\)](#) and [Chernov et al. \(2017\)](#)), implying that the costs of insurance against currency depreciation are high relative to those for currency appreciation. Indeed, in our model, states with a low shadow costs  $\Lambda_{i,t}$  are costly to insure against, and correspond to states with depressed exchange rates.

Thus, customers that have the desire to buy insurance against currency depreciation states using out of the money put options in the D2C markets will observe highly skewed quotes. By formula ([A.30](#)), we have

$$M_{i,t,t+1}^I = (\Psi_{i,t,t+1} D_{i,t,t+1})^{-1} (M_{i,t,t+1}^H)^2 (\lambda_{i,t}(S_{i,t+1}/S_{i,t}) + \mu_{i,t}), \quad (\text{A.29})$$

and therefore

$$\mathcal{E}_{i,t,t+1} = \frac{(\Psi_{i,t,t+1} D_{i,t,t+1})^{-1} (M_{i,t,t+1}^H)^2 (\lambda_{i,t}(S_{i,t+1}/S_{i,t}) + \mu_{i,t})}{(\Psi_{\$,t,t+1} D_{\$,t,t+1})^{-1} (M_{\$,t,t+1}^H)^2 (\lambda_{\$,t}(S_{\$,t+1}/S_{i,t}) + \mu_{\$,t})}. \quad (\text{A.30})$$

Thus, we arrive at the following result.

**Corollary 18** *Suppose that  $M_{i,t,t+1}^H$  stays bounded. If time  $t$  expectations lead customers into a risk-on regime so that  $\lambda_{i,t} > 0 > \mu_{i,t}$ , then, a large enough drop in the country  $i$  stock market price  $S_{i,t+1}$  at time  $t + 1$  always leads to a currency crash.*

Corollary [18](#) highlights an important boom and bust feature of currency crashes in the model. A “boom” that leads to a build up of optimistic expectations and drives customers into a “risk-on” regime leads to an endogenous build-up of risk in intermediaries’ balance sheets. In such episodes, strong drops of asset prices go hand in hand with currency crashes. This finding suggests that it may make sense to differentiate between “good” and “bad crashes”: A good crash (e.g., like the one following a dot com bubble) hits only customers,

but has no systemic implications; a bad crash hits intermediaries and therefore comes with “systemic” implications.

## B Equivalence Results: Market Power Versus Downward Sloping Demand

Suppose that each trading round  $t$  is split into two sub-periods. At time  $t-$ , customers contact intermediaries and trade state-contingent claims with them in a centralized competitive market. However, this market is subject to collateral constraints for intermediaries: they need to hold enough of liquid assets (stocks and bonds in this example) to cover their trades, and incur a regulatory cost at time  $t + 1$  (e.g., through capital requirements, leverage ratio constraints, etc), that are given by  $-K_{t+1} \log(\alpha_t^I + \beta_t^I S_{t+1} - X_{t+1})$ . Here, the cost factor  $K_{t+1}$  accounts for the fact that regulatory requirements and/or the impact of these requirements in the intermediary balance sheets can be time varying. We also assume that these firms are short-lived. Then, the maximization problem of the is given by

$$\begin{aligned} \max_{X, \alpha_{t-}^I, \beta_{t-}^I, Y_{t+, t+1}} & \left( E_t[(M_{t, t+1}^H - M_{t, t+1}^I)(X_{t+1} - \alpha_t^I - \beta_t^I S_{t+1})] \right. \\ & \left. + E_t[M_{t, t+1}^I X_{t+1}] + E_t[M_{t, t+1}^I K_{t+1} \log(\alpha_t^I + \beta_t^I S_{t+1} - X_{t+1})] - E_t[M_{t, t+1}^I Y_{t+, t+1}] + E_t[M_{t, t+1}^I Y_{t+, t+1}] \right) \end{aligned}$$

where  $\alpha_t^I, \beta_t^I$  are arbitrary and satisfy that the market price of the claim,  $E_t[M_{t, t+1}^H(\alpha_t^I + \beta_t^I S_{t+1})] \leq W_t^I$ , where  $W_t^I$  is intermediary wealth. Note that we are assuming that the time  $t+$  market is free from any collateral constraints, and hence the choice of collateral  $\alpha_t^I, \beta_t^I$  has no impact on the choice of the claim  $Y_{t+, t+1}$  traded in the D2D market. Also, we assume that this claim imposes no regulatory costs on the firm. Thus, its choice is irrelevant. We assume that the  $I$  agents can also trade stocks and bonds at both time  $t-$  and  $t$ , but they



incur no regulatory cost, and thus can perfectly arbitrage away any price discrepancies. As a result, stocks and bonds are priced fairly across the two markets, and hence the maximization problem takes the form

$$\max_{X, \alpha_{t-}^I, \beta_{t-}^I, Y_{t+, t+1}} \left( E_t[M_{t, t+1}^H X_{t+1}] \right. \\ \left. + E_t[M_{t, t+1}^I K_{t+1} \log(\alpha_t^I + \beta_t^I S_{t+1} - X_{t+1})] - E_t[M_{t, t+1}^I Y_{t+, t+1}] + E_t[M_{t, t+1}^I Y_{t+, t+1}] \right).$$

Clearly, the optimal choice always satisfies  $E_t[M_{t, t+1}^H(\alpha_t^I + \beta_t^I S_{t+1})] = W_t^I$ . The first order condition gives

$$M_{t, t+1}^H = M_{t, t+1}^I K_{t+1} (\alpha_t^I + \beta_t^I S_{t+1} - X_{t+1})^{-1},$$

while we know that  $X_{t+1} = W_t^H \Psi_{t, t+1} D_{t, t+1} (M_{t, t+1}^H)^{-1}$ . Substituting, we get

$$M_{t, t+1}^H = M_{t, t+1}^I K_{t+1} (\alpha_t^I + \beta_t^I S_{t+1} - W_t^H \Psi_{t, t+1} D_{t, t+1} (M_{t, t+1}^H)^{-1})^{-1},$$

which gives

$$M_{t, t+1}^H = \frac{W_t^H \Psi_{t, t+1} D_{t, t+1} + M_{t, t+1}^I K_{t+1}}{\alpha_t^I + \beta_t^I S_{t+1}}$$

Importantly, as in the markups case, the D2C pricing kernel explodes when  $\alpha_t^I + \beta_t^I S_{t+1}$  goes to zero because the intermediary is not willing to provide insurance against states in which the value of collateral deteriorates.

## C Proofs

**Proof of Lemma 3.** The customer rationally anticipates that he will be consuming as in formula (6): given the time  $t + 1$  wealth  $W_{i,t+1}$ , the agent will consume according to

$$C_{i,t+\tau} = \frac{W_{i,t+1}}{D_{i,t+1}} \Psi_{i,t+1,t+\tau} M_{H,t+1,t+\tau}^{-1}, \quad \tau \in [1, \dots, T - t].$$

Therefore, the agent's future value function is given by

$$U_{t+1}(W_{i,t+1}) = E_{t+1} \left[ \sum_{\tau=1}^{T-t} \Psi_{i,t+1,t+\tau} \log C_{i,t+\tau} \right] = D_{i,t+1} \log W_{i,t+1} + \text{Const}_{i,t+1}.$$

Thus, the optimization problem of the customer as a function of the quoted pricing kernel  $M_{H,t,t+1}$  takes the form

$$U_{i,t}(W_{i,t}, M_{H,t,t+1}) = \max_{W_{i,t+1}} (\log(W_{i,t} - E_t[M_{H,t,t+1}W_{i,t+1}]) + E_t[\Psi_{i,t,t+1}U_{t+1}(W_{i,t+1})])$$

and the first order condition implies

$$C_{i,t}^{-1} M_{H,t,t+1} = \Psi_{i,t,t+1} D_{i,t+1} W_{i,t+1}^{-1}$$

and hence

$$W_{i,t+1} = \Psi_{i,t,t+1} D_{i,t+1} C_{i,t} M_{H,t,t+1}^{-1} = \Psi_{i,t,t+1} D_{i,t+1} W_{i,t} D_{i,t}^{-1} M_{H,t,t+1}^{-1}.$$

Q.E.D.

In the autarky case, we just need to solve for the D2C and D2D pricing kernels in a given country. For this reason, to simplify the notation, we will omit the country index everywhere in this appendix.

**Proof of Proposition 4.** For simplicity, in this proof we omit the country index  $i$ . Suppose first that  $\mu_t > 0$ . Define  $\hat{\lambda}_t = \lambda_t/\mu_t$ . Then, we need to solve the system

$$\begin{aligned} E_t \left[ \frac{(\Psi_{t,t+1} D_{t,t+1})^{1/2} M_{I,t,t+1}^{1/2}}{(\hat{\lambda}_t(S_{t+1}/S_t) + 1)^{1/2} \mu_t^{1/2}} \right] &= E_t[M_{I,t,t+1}]; \\ E_t \left[ \frac{(\Psi_{t,t+1} D_{t,t+1})^{1/2} M_{I,t,t+1}^{1/2} S_{t+1}}{(\hat{\lambda}_t(S_{t+1}/S_t) + 1)^{1/2} \mu_t^{1/2}} \right] &= E_t[M_{I,t,t+1} S_{t+1}]. \end{aligned} \quad (\text{C.1})$$

The first equation gives  $\mu_t^{1/2} = E_t \left[ \frac{(\Psi_{t,t+1} D_{t,t+1})^{1/2} M_{I,t,t+1}^{1/2}}{(\hat{\lambda}_t(S_{t+1}/S_t) + 1)^{1/2}} \right] / E_t[M_{I,t,t+1}]$ , and, substituting into the second equation, we get

$$E_t \left[ \frac{(\Psi_{t,t+1} D_{t,t+1})^{1/2} M_{I,t,t+1}^{1/2} S_{t+1}}{(\hat{\lambda}_t(S_{t+1}/S_t) + 1)^{1/2}} \right] \frac{E_t[M_{I,t,t+1}]}{E_t \left[ \frac{(\Psi_{t,t+1} D_{t,t+1})^{1/2} M_{I,t,t+1}^{1/2}}{(\hat{\lambda}_t(S_{t+1}/S_t) + 1)^{1/2}} \right]} = E_t[M_{I,t,t+1} S_{t+1}] \quad (\text{C.2})$$

By direct calculation, the left-hand side of (C.2) is monotone decreasing in  $\hat{\lambda}_t$ . When  $\hat{\lambda}_t = 0$ , the left-hand side of (C.2) becomes

$$E_t \left[ (\Psi_{t,t+1} D_{t,t+1})^{1/2} M_{I,t,t+1}^{1/2} S_{t+1} \right] \frac{E_t[M_{I,t,t+1}]}{E_t \left[ (\Psi_{t,t+1} D_{t,t+1})^{1/2} M_{I,t,t+1}^{1/2} \right]}.$$

When  $\hat{\lambda}_t$  converges to  $+\infty$ , the left-hand side of (C.2) converges to

$$E_t \left[ \frac{(\Psi_{t,t+1} D_{t,t+1})^{1/2} M_{I,t,t+1}^{1/2} S_{t+1}}{(S_{t+1}/S_t)^{1/2}} \right] \frac{E_t[M_{I,t,t+1}]}{E_t \left[ \frac{(\Psi_{t,t+1} D_{t,t+1})^{1/2} M_{I,t,t+1}^{1/2}}{(S_{t+1}/S_t)^{1/2}} \right]},$$

while when  $\hat{\lambda}_t$  converges to its minimal possible negative value, the left-hand side of (C.2) converges to  $\max(S_{t+1}) E_t[M_{I,t,t+1}]$ . Thus, we need to consider three scenarios: if  $\max(S_{t+1}) > \frac{E_t[M_{I,t,t+1} S_{t+1}]}{E_t[M_{I,t,t+1}]}$  then there exists a  $\hat{\lambda}_t < 0$  satisfying (C.1). This

is equivalent to

$$\tilde{E}_t[S_{t+1}] > \frac{E_t \left[ (\Psi_{t,t+1} D_{t,t+1})^{1/2} M_{I,t,t+1}^{-1/2} S_{t+1} \right]}{E_t \left[ (\Psi_{t,t+1} D_{t,t+1})^{1/2} M_{I,t,t+1}^{-1/2} \right]},$$

where  $\tilde{E}$  is the expectation under the D2D risk neutral measure. If however

$$\begin{aligned} & \frac{E_t \left[ (\Psi_{t,t+1} D_{t,t+1})^{1/2} M_{I,t,t+1}^{1/2} S_{t+1} \right]}{E_t \left[ (\Psi_{t,t+1} D_{t,t+1})^{1/2} M_{I,t,t+1}^{1/2} \right]} > \frac{E_t[M_{I,t,t+1} S_{t+1}]}{E_t[M_{I,t,t+1}]} \\ & > E_t \left[ \frac{(\Psi_{t,t+1} D_{t,t+1})^{1/2} M_{I,t,t+1}^{1/2} S_{t+1}}{(S_{t+1}/S_t)^{1/2}} \right] \frac{1}{E_t \left[ \frac{(\Psi_{t,t+1} D_{t,t+1})^{1/2} M_{I,t,t+1}^{1/2}}{(S_{t+1}/S_t)^{1/2}} \right]}, \end{aligned}$$

then there exists a unique positive  $\hat{\lambda}_t$ . Finally, if

$$E_t \left[ \frac{(\Psi_{t,t+1} D_{t,t+1})^{1/2} M_{I,t,t+1}^{1/2} S_{t+1}}{(S_{t+1}/S_t)^{1/2}} \right] \frac{1}{E_t \left[ \frac{(\Psi_{t,t+1} D_{t,t+1})^{1/2} M_{I,t,t+1}^{1/2}}{(S_{t+1}/S_t)^{1/2}} \right]} > \frac{E_t[M_{I,t,t+1} S_{t+1}]}{E_t[M_{I,t,t+1}]},$$

then  $\mu_t$  needs to be negative. In this case, slightly abusing the notation, we will use  $\mu_t$  to denote  $-\mu_t$ , so that we can rewrite the system as

$$\begin{aligned} E_t \left[ \frac{(\Psi_{t,t+1} D_{t,t+1})^{1/2} M_{I,t,t+1}^{1/2}}{(\hat{\lambda}_t(S_{t+1}/S_t) - 1)^{1/2} \mu_t^{1/2}} \right] &= E_t[M_{I,t,t+1}]; \\ E_t \left[ \frac{(\Psi_{t,t+1} D_{t,t+1})^{1/2} M_{I,t,t+1}^{1/2} S_{t+1}}{(\hat{\lambda}_t(S_{t+1}/S_t) - 1)^{1/2} \mu_t^{1/2}} \right] &= E_t[M_{I,t,t+1} S_{t+1}], \end{aligned}$$

and we need to show that there is a unique positive solution  $\hat{\lambda}_t$  to

$$E_t \left[ \frac{(\Psi_{t,t+1} D_{t,t+1})^{1/2} M_{I,t,t+1}^{1/2} S_{t+1}}{(\hat{\lambda}_t(S_{t+1}/S_t) - 1)^{1/2}} \right] \frac{1}{E_t \left[ \frac{(\Psi_{t,t+1} D_{t,t+1})^{1/2} M_{I,t,t+1}^{1/2}}{(\hat{\lambda}_t(S_{t+1}/S_t) - 1)^{1/2}} \right]} = \frac{E_t[M_{I,t,t+1} S_{t+1}]}{E_t[M_{I,t,t+1}]}.$$

When  $\hat{\lambda}_t \rightarrow +\infty$ , the left-hand side converges to

$$E_t \left[ \frac{(\Psi_{t,t+1} D_{t,t+1})^{1/2} M_{I,t,t+1}^{1/2} S_{t+1}}{(S_{t+1}/S_t)^{1/2}} \right] \frac{1}{E_t \left[ \frac{(\Psi_{t,t+1} D_{t,t+1})^{1/2} M_{I,t,t+1}^{1/2}}{(S_{t+1}/S_t)^{1/2}} \right]},$$

while it converges to  $\min S_{t+1}$  when  $\hat{\lambda}_t \rightarrow S_t / \min S_{t+1}$ , and hence there is always a positive solution  $\hat{\lambda}_t$ . Q.E.D.

**Proof of Theorem 8.** We can rewrite market clearing as

$$\begin{aligned} & ((1 - \beta_k) C_{k,0}^H \Psi_{k,0,t} (M_{k,0,t}^H)^{-1} + (1 - \beta_k) C_{k,0}^I \Psi_{k,0,t} (M_{k,0,t}^I)^{-1}) \mathcal{E}_{k,t} \\ & + \bar{\theta}_k \sum_j \beta_j (C_{j,0}^H \Psi_{j,0,t} (M_{j,0,t}^H)^{-1} + C_{j,0}^I \Psi_{j,0,t} (M_{j,0,t}^I)^{-1}) \mathcal{E}_{j,t} = \mathcal{E}_{k,t} \mathcal{M}_{k,t}. \end{aligned}$$

Thus,

$$\begin{aligned} M_{k,0,t}^H &= (\mathcal{E}_{k,t} \mathcal{M}_{k,t})^{-1} M_{k,0,t}^H \left( ((1 - \beta_k) C_{k,0}^H \Psi_{k,0,t} (M_{k,0,t}^H)^{-1} + (1 - \beta_k) C_{k,0}^I \Psi_{k,0,t} (M_{k,0,t}^I)^{-1}) \mathcal{E}_{k,t} \right. \\ & \left. + \bar{\theta}_k \sum_j \beta_j (C_{j,0}^H \Psi_{j,0,t} (M_{j,0,t}^H)^{-1} + C_{j,0}^I \Psi_{j,0,t} (M_{j,0,t}^I)^{-1}) \mathcal{E}_{j,t} \right) \\ &= \mathcal{M}_{k,t}^{-1} \left( (1 - \beta_k) C_{k,0}^H \Psi_{k,0,t} + (1 - \beta_k) C_{k,0}^I \Psi_{k,0,t} (M_{k,0,t}^H / M_{k,0,t}^I) \right) \\ &+ (\mathcal{E}_{k,t} \mathcal{M}_{k,t})^{-1} M_{k,0,t}^H \bar{\theta}_k \sum_j \beta_j (C_{j,0}^H \Psi_{j,0,t} (M_{j,0,t}^H)^{-1} + C_{j,0}^I \Psi_{j,0,t} (M_{j,0,t}^I)^{-1}) \mathcal{E}_{j,t}. \end{aligned}$$

Let us make an Ansatz

$$M_{i,0,t}^J \approx M_{i,0,t}^{J,*} (1 + M_{i,0,t}^{J,(1)})$$

and recall that

$$\mathcal{E}_{j,t} = \frac{M_{i,0,t}^I}{M_{\$,0,t}^I} \approx \mathcal{E}_{j,t}^* (1 + \mathcal{E}_{j,t}^{(1)})$$

with

$$\mathcal{E}_{j,t}^{(1)} = M_{i,0,t}^{I,(1)} - M_{\$,0,t}^{I,(1)}.$$

Recall that

$$M_{k,0,t}^{I,*} = M_{k,0,t}^* = C_{k,0} \Psi_{k,0,t} \mathcal{M}_{k,t}^{-1}.$$

Thus,

$$\begin{aligned} & M_{k,0,t}^* (1 + M_{k,0,t}^{H,(1)} + M_{k,0,t}^{H,(2)}) \\ & \approx \mathcal{M}_{k,t}^{-1} \left( (1 - \beta_k) C_{k,0}^H \Psi_{k,0,t} \right. \\ & + (1 - \beta_k) C_{k,0}^I \Psi_{k,0,t} (1 + M_{k,0,t}^{H,(1)} - M_{k,0,t}^{I,(1)} + M_{k,0,t}^{H,(2)} - M_{k,0,t}^{I,(2)} + (M_{k,0,t}^{I,(1)})^2) \\ & + (\mathcal{E}_{k,t}^* \mathcal{M}_{k,t})^{-1} M_{k,0,t}^{H,*} (1 + M_{k,0,t}^{H,(1)} - \mathcal{E}_{k,t}^{(1)}) \bar{\theta}_k \\ & \times \sum_j \beta_j \left( C_{j,0}^H \Psi_{j,0,t} (M_{j,0,t}^{H,*})^{-1} (1 - M_{j,0,t}^{H,(1)}) + C_{j,0}^I \Psi_{j,0,t} (M_{j,0,t}^{I,*})^{-1} (1 - M_{j,0,t}^{I,(1)}) \right) \mathcal{E}_{j,t}^* (1 + \mathcal{E}_{j,t}^{(1)}) \\ & \approx \mathcal{M}_{k,t}^{-1} \Psi_{k,0,t} \left( (1 - \beta_k) C_{k,0}^H + (1 - \beta_k) C_{k,0}^I (1 + M_{k,0,t}^{H,(1)} - M_{k,0,t}^{I,(1)} + M_{k,0,t}^{H,(2)} - M_{k,0,t}^{I,(2)} + (M_{k,0,t}^{I,(1)})^2) \right) \\ & + M_{k,0,t}^* (1 + M_{k,0,t}^{H,(1)} - \mathcal{E}_{k,t}^{(1)}) \bar{\theta}_k \\ & \times \sum_j \beta_j C_{j,0}^{-1} \left( C_{j,0}^H (1 - M_{j,0,t}^{H,(1)}) + C_{j,0}^I (1 - M_{j,0,t}^{I,(1)}) \right) \mathcal{E}_{j,0} \frac{C_{j,0} \Psi_{j,0,t}}{C_{k,0} \Psi_{k,0,t}} (1 + \mathcal{E}_{j,t}^{(1)}) \end{aligned}$$

Dividing by  $M_{k,0,t}^*$ , we get

$$\begin{aligned}
& M_{k,0,t}^{H,(1)} + M_{k,0,t}^{H,(2)} \\
& \approx C_{k,0}^{-1} \left( -\beta_k C_{k,0}^H - C_{k,0}^I + (1 - \beta_k) C_{k,0}^I (1 + M_{k,0,t}^{H,(1)} - M_{k,0,t}^{I,(1)} + M_{k,0,t}^{H,(2)} - M_{k,0,t}^{I,(2)} + (M_{k,0,t}^{I,(1)})^2) \right) \\
& + (1 + M_{k,0,t}^{H,(1)} - \mathcal{E}_{k,t}^{(1)}) \bar{\theta}_k \\
& \times \sum_j \beta_j C_{j,0}^{-1} \left( C_{j,0}^H (1 - M_{j,0,t}^{H,(1)}) + C_{j,0}^I (1 - M_{j,0,t}^{I,(1)}) \right) \mathcal{E}_{j,0} \frac{C_{j,0} \Psi_{j,0,t}}{C_{k,0} \Psi_{k,0,t}} (1 + \mathcal{E}_{j,t}^{(1)})
\end{aligned}$$

First, we write down the system for the first order corrections:

$$M_{k,0,t}^{H,(1)} = C_{k,0}^{-1} \left( -\beta_k C_{k,0} + C_{k,0}^I (M_{k,0,t}^{H,(1)} - M_{k,0,t}^{I,(1)}) \right) + \bar{\theta}_k \sum_j \beta_j \mathcal{E}_{j,0} \frac{C_{j,0} \Psi_{j,0,t}}{C_{k,0} \Psi_{k,0,t}} \quad (\text{C.3})$$

Denote

$$\Xi_t \equiv \sum_j \beta_j C_{j,0} \mathcal{E}_{j,0} \Psi_{j,0,t}.$$

Then,

$$M_{k,t,\tau}^H \approx \mathcal{N}_{t,\tau}^{-1} \Psi_{k,t,\tau} \left( (M_{k,t,\tau}^{H,(1)} - M_{k,t,\tau}^{I,(1)}) + \theta_k (\Xi_\tau \Psi_{k,0,\tau}^{-1} - \Xi_t \Psi_{k,0,t}^{-1}) \right)$$

Note that

$$\Delta C_{t,\tau}^{H/k,*} = \Xi_\tau \Psi_{k,0,\tau}^{-1} - \Xi_t \Psi_{k,0,t}^{-1}.$$

At the same time,

$$\begin{aligned}
M_{k,t,t+1}^I &= (M_{k,t,t+1}^H)^2 (\Psi_{k,t,t+1} D_{k,t,t+1})^{-1} (\lambda_{k,t} (S_{k,t+1}/S_{k,t}) + \mu_{k,t}) \\
&\approx \mathcal{N}_{k,t+1}^{-2} (1 + 2(M_{k,t,t+1}^{H,(1)} + M_{k,t,t+1}^{H,(2)} + (M_{k,t,t+1}^{H,(1)})^2) \Psi_{k,t,t+1} (D_{k,t,t+1})^{-1} \\
&\times ((1 + \lambda_{k,t}^{(1)} + \lambda_{k,t}^{(2)}) \mathcal{N}_{k,t+1} D_{k,t,t+1} (1 + S_{k,t,t+1}^{(1)} + S_{k,t,t+1}^{(2)} + \mu_{k,t}^{(1)} + \mu_{k,t}^{(2)}) \\
&\approx M_{k,t,t+1}^{I,*} (1 + 2(M_{k,t,t+1}^{H,(1)} + M_{k,t,t+1}^{H,(2)} + (M_{k,t,t+1}^{H,(1)})^2) \\
&\times \left( 1 + (\lambda_{k,t}^{(1)} + S_{k,t,t+1}^{(1)} + \mu_{k,t}^{(1)} (\mathcal{N}_{k,t+1} D_{k,t,t+1})^{-1}) + (\lambda_{k,t}^{(2)} + \lambda_{k,t}^{(1)} S_{k,t,t+1}^{(1)} + S_{k,t,t+1}^{(2)} + \mu_{k,t}^{(2)} (\mathcal{N}_{k,t+1} D_{k,t,t+1})^{-1}) \right) \\
&\approx M_{k,t,t+1}^{I,*} \left( 1 + 2(M_{k,t,t+1}^{H,(1)} + (\lambda_{k,t}^{(1)} + S_{k,t,t+1}^{(1)} + \mu_{k,t}^{(1)} (\mathcal{N}_{k,t+1} D_{k,t,t+1})^{-1}) \right. \\
&+ \lambda_{k,t}^{(2)} + \lambda_{k,t}^{(1)} S_{k,t,t+1}^{(1)} + S_{k,t,t+1}^{(2)} + \mu_{k,t}^{(2)} (\mathcal{N}_{k,t+1} D_{k,t,t+1})^{-1} \\
&+ 2M_{k,t,t+1}^{H,(1)} (\lambda_{k,t}^{(1)} + S_{k,t,t+1}^{(1)} + \mu_{k,t}^{(1)} (\mathcal{N}_{k,t+1} D_{k,t,t+1})^{-1}) + 2M_{k,t,t+1}^{H,(2)} + (M_{k,t,t+1}^{H,(1)})^2 \left. \right).
\end{aligned} \tag{C.4}$$

Now, using that  $S_{k,t}$  is priced correctly both under the D2C and D2D kernels and iterating the identity

$$S_{k,t} = \mathcal{M}_{k,t} + E_t[M_{t,t+1}^H S_{k,t+1}],$$

we get

$$S_{k,t} \approx \mathcal{M}_{k,t} D_{k,t} (1 + \bar{\theta}_k (\bar{W}_t^{H/k,*} - (\bar{\Psi}_t / \Psi_{k,t}))),$$

where we have defined

$$\bar{W}_t \equiv \sum_j \beta_j C_{j,0} \mathcal{E}_{j,0} \Psi_{j,0,t} D_{j,t} = \mathcal{M}_{\$,t}^{-1} C_{\$,0} \Psi_{\$,0,t} \sum_j \beta_j \mathcal{E}_{j,t} S_{j,t}^*$$



and

$$\bar{W}_t^{H/k,*} \equiv \frac{\bar{W}_t}{\Psi_{k,0,t} D_{k,t}} = \frac{\bar{S}_t^\$}{S_{k,t}^\$}$$

Hence,

$$\begin{aligned} M_{k,t,t+1}^I &\approx \mathcal{N}_{k,t+1}^{-1} \Psi_{k,t,t+1} \\ &\times (1 + 2M_{k,t,t+1}^{H,(1)} - \bar{\theta}_k \Delta \bar{C}_{t,t+1}^{H/k,*} + \bar{\theta}_k \Delta \bar{W}_{t,t+1}^{H/k,*} + \lambda_{k,t}^{(1)} + \mu_{k,t}(\mathcal{N}_{k,t+1} D_{k,t,t+1}^H)^{-1}) \end{aligned}$$

and therefore

$$\begin{aligned} M_{k,t,t+1}^{H,(1)} &= C_{k,0}^{-1} C_{k,0}^I (M_{k,t,t+1}^{H,(1)} - M_{k,t,t+1}^{I,(1)}) + \bar{\theta}_k \Delta \bar{C}_{t,t+1}^{H/k,*} \\ &= C_{k,0}^{-1} C_{k,0}^I (-M_{k,t,t+1}^{H,(1)} + \bar{\theta}_k \Delta \bar{C}_{t,t+1}^{H/k,*} - \bar{\theta}_k \Delta \bar{W}_{t,t+1}^{H/k,*} - \lambda_{k,t}^{(1)} - \mu_{k,t}(\mathcal{N}_{k,t+1} D_{k,t,t+1}^H)^{-1}) + \bar{\theta}_k \Delta \bar{C}_{t,t+1}^{H/k,*} \end{aligned} \quad (\text{C.5})$$

and hence

$$M_{k,t,t+1}^{H,(1)} = \bar{\theta}_k \Delta \bar{C}_{t,t+1}^{H/k,*} - \frac{C_{k,0}^I}{2C_{k,0}^I + C_{k,0}^H} (\bar{\theta}_k \Delta \bar{W}_{t,t+1}^{H/k,*} + \lambda_{k,t}^{(1)} + \mu_{k,t}(\mathcal{N}_{k,t+1} D_{k,t,t+1}^H)^{-1}).$$

Therefore,

$$\begin{aligned} M_{k,t,t+1}^{I,(1)} &= 2M_{k,t,t+1}^{H,(1)} - \bar{\theta}_k \Delta \bar{C}_{t,t+1}^{H/k,*} + \bar{\theta}_k \Delta \bar{W}_{t,t+1}^{H/k,*} + \lambda_{k,t}^{(1)} + \mu_{k,t}(\mathcal{N}_{k,t+1} D_{k,t,t+1}^H)^{-1} \\ &= 2\bar{\theta}_k \Delta \bar{C}_{t,t+1}^{H/k,*} - 2\frac{C_{k,0}^I}{2C_{k,0}^I + C_{k,0}^H} (\bar{\theta}_k \Delta \bar{W}_{t,t+1}^{H/k,*} + \lambda_{k,t}^{(1)} + \mu_{k,t}(\mathcal{N}_{k,t+1} D_{k,t,t+1}^H)^{-1}) \\ &\quad - \bar{\theta}_k \Delta \bar{C}_{t,t+1}^{H/k,*} + \bar{\theta}_k \Delta \bar{W}_{t,t+1}^{H/k,*} + \lambda_{k,t}^{(1)} + \mu_{k,t}(\mathcal{N}_{k,t+1} D_{k,t,t+1}^H)^{-1} \\ &= \bar{\theta}_k \Delta \bar{C}_{t,t+1}^{H/k,*} + \frac{C_{k,0}^H}{2C_{k,0}^I + C_{k,0}^H} (\bar{\theta}_k \Delta \bar{W}_{t,t+1}^{H/k,*} + \lambda_{k,t}^{(1)} + \mu_{k,t}(\mathcal{N}_{k,t+1} D_{k,t,t+1}^H)^{-1}) \end{aligned}$$

and

$$\begin{aligned}
M_{k,t,t+1}^{H,(1)} &- M_{k,t,t+1}^{I,(1)} \\
&= -M_{k,t,t+1}^{H,(1)} + \bar{\theta}_k \Delta \bar{C}_{t,t+1}^{H/k,*} - \bar{\theta}_k \Delta \bar{W}_{t,t+1}^{H/k,*} - \lambda_{k,t}^{(1)} - \mu_{k,t} (\mathcal{N}_{k,t+1} D_{k,t,t+1}^H)^{-1} \\
&= -(\bar{\theta}_k \Delta \bar{C}_{t,t+1}^{H/k,*} - \frac{C_{k,0}^I}{2C_{k,0}^I + C_{k,0}^H} (\bar{\theta}_k \Delta \bar{W}_{t,t+1}^{H/k,*} + \lambda_{k,t}^{(1)} + \mu_{k,t} (\mathcal{N}_{k,t+1} D_{k,t,t+1}^H)^{-1})) \\
&\quad + \bar{\theta}_k \Delta \bar{C}_{t,t+1}^{H/k,*} - \bar{\theta}_k \Delta \bar{W}_{t,t+1}^{H/k,*} - \lambda_{k,t}^{(1)} - \mu_{k,t} (\mathcal{N}_{k,t+1} D_{k,t,t+1}^H)^{-1} \\
&= -\frac{C_{k,0}^I + C_{k,0}^H}{2C_{k,0}^I + C_{k,0}^H} (\bar{\theta}_k \Delta \bar{W}_{t,t+1}^{H/k,*} + \lambda_{k,t}^{(1)} + \mu_{k,t} (\mathcal{N}_{k,t+1} D_{k,t,t+1}^H)^{-1})
\end{aligned} \tag{C.6}$$

Therefore, the equations for the Lagrange multipliers are

$$\begin{aligned}
E_t[M_{k,t,t+1}^* (1 + M_{t,t+1}^{H,(1)} + M_{t,t+1}^{H,(2)})] &= E_t[M_{k,t,t+1}^* (1 + M_{t,t+1}^{I,(1)} + M_{t,t+1}^{I,(2)})] \\
E_t[M_{k,t,t+1}^* (1 + M_{t,t+1}^{H,(1)} + M_{t,t+1}^{H,(2)}) \mathcal{M}_{k,t+1} D_{k,t+1} (1 + S_{k,t+1}^{(1)} + S_{k,t+1}^{(2)})] & \\
&= E_t[M_{k,t,t+1}^* (1 + M_{t,t+1}^{I,(1)} + M_{t,t+1}^{I,(2)}) \mathcal{M}_{k,t+1} D_{k,t+1} (1 + S_{k,t+1}^{(1)} + S_{k,t+1}^{(2)})].
\end{aligned} \tag{C.7}$$

To the first order, this gives

$$\begin{aligned}
E_t[M_{k,t,t+1}^* M_{t,t+1}^{H,(1)}] &= E_t[M_{k,t,t+1}^* M_{t,t+1}^{I,(1)}] \\
E_t[M_{k,t,t+1}^* \mathcal{M}_{k,t+1} D_{k,t+1} (M_{t,t+1}^{H,(1)} + S_{k,t+1}^{(1)})] &= E_t[M_{k,t,t+1}^* \mathcal{M}_{k,t+1} D_{k,t+1} (M_{t,t+1}^{I,(1)} + S_{k,t+1}^{(1)})],
\end{aligned}$$

which can be rewritten as

$$\begin{aligned}
E_t[M_{k,t,t+1}^* (M_{t,t+1}^{H,(1)} - M_{t,t+1}^{I,(1)})] &= 0 \\
E_t[M_{k,t,t+1}^* \mathcal{M}_{k,t+1} D_{k,t+1} (M_{t,t+1}^{H,(1)} - M_{t,t+1}^{I,(1)})] &= 0.
\end{aligned}$$

Substituting the expression for the difference in pricing kernel corrections, we get

$$\begin{aligned}
E_t[\mathcal{N}_{k,t+1}^{-1} \Psi_{k,t,t+1} (\bar{\theta}_k \Delta \bar{W}_{t,t+1}^{H/k,*} + \lambda_{k,t}^{(1)} + \mu_{k,t} (\mathcal{N}_{k,t+1} D_{k,t,t+1}^H)^{-1})] &= 0 \\
E_t[\mathcal{N}_{k,t+1}^{-1} \Psi_{k,t,t+1} \mathcal{N}_{k,t+1} D_{k,t,t+1} (\bar{\theta}_k \Delta \bar{W}_{t,t+1}^{H/k,*} + \lambda_{k,t}^{(1)} + \mu_{k,t} (\mathcal{N}_{k,t+1} D_{k,t,t+1}^H)^{-1})] &= 0.
\end{aligned}$$

and the claim follows in complete analogy with formula (C.12).

Q.E.D.

**Proof of Proposition 15.** We now go to the second of order. In this case, we get from (C.5) that

$$\begin{aligned}
& M_{k,0,t}^{H,(2)} \\
& \approx C_{k,0}^{-1} \left( C_{k,0}^I (M_{k,0,t}^{H,(2)} - M_{k,0,t}^{I,(2)} + (M_{k,0,t}^{I,(1)})^2) - \beta_k C_{k,0}^I (1 + M_{k,0,t}^{H,(1)} - M_{k,0,t}^{I,(1)}) \right) \\
& + \bar{\theta}_k \sum_j \beta_j C_{j,0}^{-1} \mathcal{E}_{j,0} \frac{C_{j,0} \Psi_{j,0,t}}{C_{k,0} \Psi_{k,0,t}} \\
& \times \left( C_{j,0}^H (\mathcal{E}_{j,t}^{(1)} - M_{j,0,t}^{H,(1)} + M_{k,0,t}^{H,(1)} - \mathcal{E}_{k,t}^{(1)}) + C_{j,0}^I (M_{k,0,t}^{H,(1)} - \mathcal{E}_{k,t}^{(1)} + \mathcal{E}_{j,t}^{(1)} - M_{j,0,t}^{I,(1)}) \right) \\
& = C_{k,0}^{-1} \left( C_{k,0}^I (M_{k,0,t}^{H,(2)} - M_{k,0,t}^{I,(2)} + (M_{k,0,t}^{I,(1)})^2) - \beta_k C_{k,0}^I (M_{k,0,t}^{H,(1)} - M_{k,0,t}^{I,(1)}) \right) \\
& + \bar{\theta}_k \sum_j \beta_j C_{j,0}^{-1} \mathcal{E}_{j,0} \frac{C_{j,0} \Psi_{j,0,t}}{C_{k,0} \Psi_{k,0,t}} \\
& \times \left( C_{j,0}^H (M_{j,0,t}^{I,(1)} - M_{k,0,t}^{I,(1)} - M_{j,0,t}^{H,(1)} + M_{k,0,t}^{H,(1)}) + C_{j,0}^I (M_{k,0,t}^{H,(1)} + M_{j,0,t}^{I,(1)} - M_{k,0,t}^{I,(1)} - M_{j,0,t}^{I,(1)}) \right) \\
& = C_{k,0}^{-1} \left( C_{k,0}^I (M_{k,0,t}^{H,(2)} - M_{k,0,t}^{I,(2)} + (M_{k,0,t}^{I,(1)})^2) - \beta_k C_{k,0}^I (M_{k,0,t}^{H,(1)} - M_{k,0,t}^{I,(1)}) \right) \\
& + \bar{\theta}_k \sum_j \beta_j C_{j,0}^{-1} \mathcal{E}_{j,0} \frac{C_{j,0} \Psi_{j,0,t}}{C_{k,0} \Psi_{k,0,t}} \\
& \times \left( C_{j,0}^H (-(M_{j,0,t}^{H,(1)} - M_{j,0,t}^{I,(1)}) + (M_{k,0,t}^{H,(1)} - M_{k,0,t}^{I,(1)})) + C_{j,0}^I (M_{k,0,t}^{H,(1)} - M_{k,0,t}^{I,(1)}) \right).
\end{aligned}$$

Rewriting, we get

$$\begin{aligned}
M_{k,0,t}^{H,(2)} &= C_{k,0}^{-1} C_{k,0}^I (M_{k,0,t}^{H,(2)} - M_{k,0,t}^{I,(2)}) + C_{k,0}^{-1} C_{k,0}^I (M_{k,0,t}^{I,(1)})^2 \\
&+ (M_{k,0,t}^{H,(1)} - M_{k,0,t}^{I,(1)}) \left( -\beta_k C_{k,0}^{-1} C_{k,0}^I + \bar{\theta}_k \sum_j \beta_j \mathcal{E}_{j,0} \frac{C_{j,0} \Psi_{j,0,t}}{C_{k,0} \Psi_{k,0,t}} \right) \\
&- \bar{\theta}_k \sum_j \beta_j C_{j,0}^{-1} \mathcal{E}_{j,0} \frac{C_{j,0} \Psi_{j,0,t}}{C_{k,0} \Psi_{k,0,t}} C_{j,0}^H (M_{j,0,t}^{H,(1)} - M_{j,0,t}^{I,(1)}) \\
&= C_{k,0}^{-1} C_{k,0}^I (M_{k,0,t}^{H,(2)} - M_{k,0,t}^{I,(2)}) + C_{k,0}^{-1} C_{k,0}^I (M_{k,0,t}^{I,(1)})^2 \\
&+ (M_{k,0,t}^{H,(1)} - M_{k,0,t}^{I,(1)}) \left( -\beta_k C_{k,0}^{-1} C_{k,0}^I + \bar{\theta}_k \bar{W}_t^{H/k,*} \right) \\
&+ \bar{\theta}_k \Xi_t^{/k}
\end{aligned}$$

where we have defined

$$\begin{aligned}
\Xi_t^{/k} &= \sum_j \beta_j C_{j,0}^{-1} \mathcal{E}_{j,0} \frac{C_{j,0} \Psi_{j,0,t}}{C_{k,0} \Psi_{k,0,t}} C_{j,0}^H (M_{j,0,t}^{H,(1)} - M_{j,0,t}^{I,(1)}) \\
&= -\sum_j \beta_j C_{j,0}^{-1} \mathcal{E}_{j,0} \frac{C_{j,0} \Psi_{j,0,t}}{C_{k,0} \Psi_{k,0,t}} C_{j,0}^H \frac{C_{j,0}^I + C_{j,0}^H}{2C_{j,0}^I + C_{j,0}^H} \bar{\theta}_j \left( \Delta \bar{W}_t^{H/j,*} + \sum_{\tau=0}^{t-1} \left( \lambda_{j,\tau}^{(1)} + \mu_{j,\tau} (\mathcal{N}_{j,\tau+1} D_{j,\tau,\tau+1}^H)^{-1} \right) \right)
\end{aligned}$$

Therefore,

$$M_{k,t,t+1}^H \approx M_{k,t,t+1}^* (1 + M_{k,t,t+1}^{(1)} + (M_{k,0,t+1}^{H,(2)} - M_{k,0,t}^{H,(2)} + (M_{k,0,t}^{H,(1)})^2))$$

and hence

$$M_{k,t,t+1}^{(2)} = M_{k,0,t+1}^{H,(2)} - M_{k,0,t}^{H,(2)} + (M_{k,0,t}^{H,(1)})^2 \quad (\text{C.8})$$

Now, the second order correction in equations (C.9) can be rewritten as

$$\begin{aligned} E_t[M_{k,t,t+1}^*(M_{t,t+1}^{H,(2)} - M_{t,t+1}^{I,(2)})] &= 0 \\ E_t[M_{k,t,t+1}^* \mathcal{N}_{k,t,t+1} D_{k,t,t+1} \left( (M_{t,t+1}^{H,(1)} - M_{t,t+1}^{I,(1)}) S_{k,t+1}^{(1)} + (M_{t,t+1}^{H,(2)} - M_{t,t+1}^{I,(2)}) \right)] &= 0. \end{aligned} \quad (\text{C.9})$$

Thus, we have, using the the second of the identities (C.9), that

$$\begin{aligned} S_{k,t} &= S_{k,t}^*(1 + S_{k,t}^{(1)}) + E_t[M_{k,t,t+1}^{H,*} S_{k,t+1}^* S_{k,t+1}^{(2)}] + E_t[M_{k,t,t+1}^{H,(1)} S_{k,t+1}^{(1)}] + E_t[M_{k,t,t+1}^{H,*} M_{k,t,t+1}^{H,(2)} S_{k,t+1}^*] \\ &= S_{k,t}^*(1 + S_{k,t}^{(1)}) + E_t[M_{k,t,t+1}^{H,*} S_{k,t+1}^* S_{k,t+1}^{(2)}] + E_t[M_{k,t,t+1}^{H,(1)} S_{k,t+1}^{(1)}] \\ &+ E_t \left[ \left( (M_{k,0,t}^{H,(1)})^2 - C_{k,0}^{-1} C_{k,0}^I (M_{t,t+1}^{H,(1)} - M_{t,t+1}^{I,(1)}) S_{k,t+1}^{(1)} \right. \right. \\ &+ C_{k,0}^{-1} C_{k,0}^I ((M_{k,0,t+1}^{I,(1)})^2 - (M_{k,0,t}^{I,(1)})^2) \\ &+ (M_{k,0,t+1}^{H,(1)} - M_{k,0,t+1}^{I,(1)}) \left( -\beta_k C_{k,0}^{-1} C_{k,0}^I + \bar{\theta}_k \bar{W}_{t+1}^{H/k,*} \right) - (M_{k,0,t}^{H,(1)} - M_{k,0,t}^{I,(1)}) \left( -\beta_k C_{k,0}^{-1} C_{k,0}^I + \bar{\theta}_k \bar{W}_t^{H/k,*} \right) \\ &\left. \left. + \bar{\theta}_k \Delta \Xi_{t,t+1}^{/k} \right) M_{k,t,t+1}^{H,*} S_{k,t+1}^* \right] \end{aligned}$$

Define

$$\begin{aligned} A_{k,t} &\equiv E_t[M_{k,t,t+1}^{H,(1)} S_{k,t+1}^{(1)}] \\ &+ E_t \left[ \left( (M_{k,0,t}^{H,(1)})^2 - C_{k,0}^{-1} C_{k,0}^I (M_{t,t+1}^{H,(1)} - M_{t,t+1}^{I,(1)}) S_{k,t+1}^{(1)} \right. \right. \\ &+ C_{k,0}^{-1} C_{k,0}^I ((M_{k,0,t+1}^{I,(1)})^2 - (M_{k,0,t}^{I,(1)})^2) \\ &+ (M_{k,0,t+1}^{H,(1)} - M_{k,0,t+1}^{I,(1)}) \left( -\beta_k C_{k,0}^{-1} C_{k,0}^I + \bar{\theta}_k \bar{W}_{t+1}^{H/k,*} \right) \\ &\left. \left. - (M_{k,0,t}^{H,(1)} - M_{k,0,t}^{I,(1)}) \left( -\beta_k C_{k,0}^{-1} C_{k,0}^I + \bar{\theta}_k \bar{W}_t^{H/k,*} \right) \right) M_{k,t,t+1}^{H,*} S_{k,t+1}^* \right] \end{aligned}$$

and note that  $A_{k,t}$  only depends on the domestic monetary policy in country  $k$  (though in a

quite complicated fashion). Then, we can rewrite the equation for  $S_{k,t}$  as

$$S_{k,t}^* S_{k,t}^{(2)} = E_t[M_{k,t,t+1}^{H,*} S_{k,t+1}^* S_{k,t+1}^{(2)}] + A_{k,t} + E_t \left[ \bar{\theta}_k \Delta \Xi_{t,t+1}^{/k} M_{k,t,t+1}^{H,*} S_{k,t+1}^* \right],$$

which defines  $S_{k,t}^{(2)}$ . Thus,

$$S_{k,t}^{(2)} = -\bar{\theta}_k \Xi_t^{/k} E_t[M_{k,t,t+1}^{H,*} S_{k,t+1}^*] / S_{k,t}^* = -\bar{\theta}_k \Xi_t^{/k} (1 - \mathcal{M}_{k,t} / S_{k,t}^*) + Z_{k,t}$$

where  $Z_{k,t}$  only depends on the domestic monetary policy as well as expectations about future policy. Now, from (C.4), we get that

$$M_{k,t,t+1}^{I,(2)} = 2M_{k,t,t+1}^{H,(2)} + S_{k,t,t+1}^{(2)} + Q_{k,t,t+1}$$

where  $Q_{k,t,t+1}$  is a (complicated) expression that only depends only on the domestic monetary policy.

Substituting into (C.11), we get

$$\begin{aligned} M_{k,t,t+1}^{H,(2)} &= C_{k,0}^{-1} C_{k,0}^I (M_{k,t,t+1}^{H,(2)} - M_{k,t,t+1}^{I,(2)}) + \tilde{Z} + \bar{\theta}_k \Delta \Xi_{t,t+1}^{/k} \\ &= C_{k,0}^{-1} C_{k,0}^I (M_{k,t,t+1}^{H,(2)} - 2M_{k,t,t+1}^{H,(2)} - S_{k,t,t+1}^{(2)} - Q_{k,t,t+1}) + \tilde{Z} + \bar{\theta}_k \Delta \Xi_{t,t+1}^{/k}, \end{aligned} \tag{C.10}$$

where  $\tilde{Z}$  does not depend on foreign monetary shocks. Hence,

$$M_{k,t,t+1}^{H,(2)} = (1 + C_{k,0}^{-1} C_{k,0}^I)^{-1} (-C_{k,0}^{-1} C_{k,0}^I S_{k,t,t+1}^{(2)} + \bar{\theta}_k \Delta \Xi_{t,t+1}^{/k}) + \hat{Z}$$

where

$$S_{k,t,t+1}^{(2)} = S_{k,t+1}^{(2)} - S_{k,t}^{(2)} + (S_{k,t}^{(1)})^2.$$

Substituting, we get

$$\begin{aligned}
M_{k,t,t+1}^{H,(2)} &= (1 + C_{k,0}^{-1}C_{k,0}^I)^{-1}(-C_{k,0}^{-1}C_{k,0}^I(S_{k,t+1}^{(2)} - S_{k,t}^{(2)} + (S_{k,t}^{(1)})^2) + \bar{\theta}_k \Delta \Xi_{t,t+1}^{/k}) + \hat{Z} \\
&= (1 + C_{k,0}^{-1}C_{k,0}^I)^{-1}(-C_{k,0}^{-1}C_{k,0}^I(-\bar{\theta}_k \Xi_{t+1}^{/k}(1 - \mathcal{M}_{k,t+1}/S_{k,t+1}^*) + \bar{\theta}_k \Xi_t^{/k}(1 - \mathcal{M}_{k,t}/S_{k,t}^*)) + \bar{\theta}_k \Delta \Xi_{t,t+1}^{/k}) + \hat{Q} \\
&= (1 + C_{k,0}^{-1}C_{k,0}^I)^{-1} \bar{\theta}_k \Xi_{t+1}^{/k} \left(1 + C_{k,0}^{-1}C_{k,0}^I(1 - \mathcal{M}_{k,t+1}/S_{k,t+1}^*)\right) \\
&\quad - (1 + C_{k,0}^{-1}C_{k,0}^I)^{-1} \bar{\theta}_k \Xi_t^{/k} \left(1 + C_{k,0}^{-1}C_{k,0}^I(1 - \mathcal{M}_{k,t}/S_{k,t}^*)\right) + Q^H,
\end{aligned} \tag{C.11}$$

where all the  $Q$  and  $Z$  terms do not depend on the foreign shocks and only depend on their expectations. Thus,

$$\begin{aligned}
M_{k,t,t+1}^{I,(2)} &= 2M_{k,t,t+1}^{H,(2)} + S_{k,t,t+1}^{(2)} + Q_{k,t,t+1} \\
&= 2(1 + C_{k,0}^{-1}C_{k,0}^I)^{-1} \bar{\theta}_k \Xi_{t+1}^{/k} \left(1 + C_{k,0}^{-1}C_{k,0}^I(1 - \mathcal{M}_{k,t+1}/S_{k,t+1}^*)\right) \\
&\quad - 2(1 + C_{k,0}^{-1}C_{k,0}^I)^{-1} \bar{\theta}_k \Xi_t^{/k} \left(1 + C_{k,0}^{-1}C_{k,0}^I(1 - \mathcal{M}_{k,t}/S_{k,t}^*)\right) \\
&\quad - \bar{\theta}_k \Xi_{t+1}^{/k}(1 - \mathcal{M}_{k,t+1}/S_{k,t+1}^*) + \bar{\theta}_k \Xi_t^{/k}(1 - \mathcal{M}_{k,t}/S_{k,t}^*) + Q^{**} \\
&= (1 + C_{k,0}^{-1}C_{k,0}^I)^{-1} \bar{\theta}_k \Xi_{t+1}^{/k} \left(2 - (1 - C_{k,0}^{-1}C_{k,0}^I)(1 - \mathcal{M}_{k,t+1}/S_{k,t+1}^*)\right) \\
&\quad - 2(1 + C_{k,0}^{-1}C_{k,0}^I)^{-1} \bar{\theta}_k \Xi_t^{/k} \left(2 - (1 - C_{k,0}^{-1}C_{k,0}^I)(1 - \mathcal{M}_{k,t}/S_{k,t}^*)\right) + Q^{***}.
\end{aligned}$$

Thus, the shock to the exchange rate  $\mathcal{E}_{i,t+1}/\mathcal{E}_{i,t}$  is given by

$$\begin{aligned}
&- \left( (1 + C_{k,0}^{-1}C_{k,0}^I)^{-1} \bar{\theta}_k \left(2 - (1 - C_{k,0}^{-1}C_{k,0}^I)(1 - \mathcal{M}_{k,t+1}/S_{k,t+1}^*)\right) \frac{1}{C_{k,0} \Psi_{k,0,t+1}} \right. \\
&\quad \left. - (1 + C_{\$,0}^{-1}C_{\$,0}^I)^{-1} \bar{\theta}_{\$} \left(2 - (1 - C_{\$,0}^{-1}C_{\$,0}^I)(1 - \mathcal{M}_{\$,t+1}/S_{\$,t+1}^*)\right) \frac{1}{C_{\$,0} \Psi_{\$,0,t+1}} \right) \\
&\quad \times \sum_j \beta_j C_{j,0}^{-1} \mathcal{E}_{j,0} C_{j,0} \Psi_{j,0,t} C_{j,0}^H \frac{C_{j,0}^I + C_{j,0}^H}{2C_{j,0}^I + C_{j,0}^H} \bar{\theta}_j \mu_{j,\tau} (\mathcal{N}_{j,\tau+1} D_{j,\tau,\tau+1}^H)^{-1}
\end{aligned}$$

Similarly, the sensitivity of relative net worth of customers in countries  $i$  and  $j$ ,  $W_{i,t+1}^H/W_{j,t+1}^H$ ,

to US monetary policy shocks  $\mathcal{N}_{i,t+1}^{-1}$  is given by

$$\begin{aligned} & \frac{C_{i,0}^H}{C_{j,0}^H} \left( (1 + C_{i,0}^{-1} C_{i,0}^I)^{-1} \bar{\theta}_i \left( 1 + C_{i,0}^{-1} C_{i,0}^I (1 - D_{i,t+1}^{-1}) \right) \frac{1}{C_{i,0} \Psi_{i,0,t}} \right. \\ & \quad \left. - (1 + C_{j,0}^{-1} C_{j,0}^I)^{-1} \bar{\theta}_j \left( 1 + C_{j,0}^{-1} C_{j,0}^I (1 - D_{j,t+1}^{-1}) \right) \frac{1}{C_{j,0} \Psi_{j,0,t}} \right) \\ & \quad \times \beta_{\$} C_{\$,0}^H \frac{C_{\$,0}^I + C_{\$,0}^H}{2C_{\$,0}^I + C_{\$,0}^H} \bar{\theta}_{\$} \mu_{\$,t} (\mathcal{N}_{\$,t+1} D_{j,t,t+1}^H)^{-1} \mathcal{E}_{\$,0} \Psi_{\$,0,t} \end{aligned}$$

Q.E.D.

**Proof of Proposition ??.** The total Dollar wealth of country  $k$  is proportional to

$$\begin{aligned} & (C_{k,0}^H \Psi_{k,0,t} (M_{k,0,t}^H)^{-1} + C_{k,0}^I \Psi_{k,0,t} (M_{k,0,t}^I)^{-1}) \mathcal{E}_{k,t} \\ & \approx (C_{k,0}^H \Psi_{k,0,t} (M_{k,0,t}^*)^{-1} (1 - M_{k,0,t}^{H,(1)}) + C_{k,0}^I \Psi_{k,0,t} (M_{k,0,t}^*)^{-1} (1 - M_{k,0,t}^{I,(1)})) \mathcal{E}_{k,t}^* (1 + M_{k,0,t}^{I,(1)} - M_{\$,0,t}^{I,(1)}) \end{aligned}$$

and therefore, using the assumed normalization  $C_{k,0}^H + C_{k,0}^I = 1$ , we get that, by (C.6), the wealth change is proportional to

$$\begin{aligned} & -C_{k,0}^H M_{k,0,t}^{H,(1)} - C_{k,0}^I M_{k,0,t}^{I,(1)} + M_{k,0,t}^{I,(1)} - M_{\$,0,t}^{I,(1)} = C_{k,0}^H (M_{k,0,t}^{I,(1)} - M_{k,0,t}^{H,(1)}) - M_{\$,0,t}^{I,(1)} \\ & = \frac{C_{k,0}^H}{2C_{k,0}^I + C_{k,0}^H} (\bar{\theta}_k \Delta \bar{W}_{t,t+1}^{H/k,*} + \lambda_{k,t}^{(1)} + \mu_{k,t} (\mathcal{N}_{k,t+1} D_{k,t,t+1}^H)^{-1}) - M_{\$,0,t}^{I,(1)}. \end{aligned}$$

Q.E.D.

The following auxiliary lemma shows that stock prices inherit the one factor structure of discount rates.

**Lemma 19** *Suppose that the transition density of  $\omega_t$  has the monotone likelihood property:*

*$\frac{\partial}{\partial \omega_t} \log p(\omega_t, \omega_{t+1})$  is strictly monotone increasing in  $\omega_{t+1}$  for almost every  $(\omega_t, \delta_{t+1})$ . Under*

*Assumption 3:*



- There exist strictly monotone increasing functions  $d_i(\omega, t)$  such that  $\log D_{i,t} = d_i(\omega_t, t)$ .<sup>74</sup>
- $S_{i,t}^{\$}/S_{j,t}^{\$}$  is monotone increasing in  $\omega_t$  if and only if  $\delta_i^{\Psi} > \delta_j^{\Psi}$ .

The proof is straightforward and follows by standard arguments.

**Proof of Proposition 9.** The equations for the Lagrange multipliers are

$$\begin{aligned}
E_t[M_{k,t,t+1}^*(1 + M_{t,t+1}^{H,(1)})] &= E_t[M_{k,t,t+1}^*(1 + M_{t,t+1}^{I,(1)})] \\
E_t[M_{k,t,t+1}^*(1 + M_{t,t+1}^{H,(1)})\mathcal{M}_{k,t+1}D_{k,t+1}(1 + S_{k,t+1}^{(1)})] \\
&= E_t[M_{k,t,t+1}^*(1 + M_{t,t+1}^{I,(1)})\mathcal{M}_{k,t+1}D_{k,t+1}(1 + S_{k,t+1}^{(1)})].
\end{aligned}$$

To the first order, this gives

$$\begin{aligned}
E_t[M_{k,t,t+1}^*M_{t,t+1}^{H,(1)}] &= E_t[M_{k,t,t+1}^*M_{t,t+1}^{I,(1)}] \\
E_t[M_{k,t,t+1}^*\mathcal{M}_{k,t+1}D_{k,t+1}(M_{t,t+1}^{H,(1)} + S_{k,t+1}^{(1)})] &= E_t[M_{k,t,t+1}^*\mathcal{M}_{k,t+1}D_{k,t+1}(M_{t,t+1}^{I,(1)} + S_{k,t+1}^{(1)})],
\end{aligned}$$

which can be rewritten as

$$\begin{aligned}
E_t[M_{k,t,t+1}^*(M_{t,t+1}^{H,(1)} - M_{t,t+1}^{I,(1)})] &= 0 \\
E_t[M_{k,t,t+1}^*\mathcal{M}_{k,t+1}D_{k,t+1}(M_{t,t+1}^{H,(1)} - M_{t,t+1}^{I,(1)})] &= 0.
\end{aligned}$$

Substituting the expression for the difference in pricing kernel corrections, we get

$$\begin{aligned}
E_t[\mathcal{N}_{k,t+1}^{-1}\Psi_{k,t,t+1}^H(\Delta W_{k,t,t+1}^{*,I/H} + \lambda_{k,t}^{(1)} + \mu_{k,t}(\mathcal{N}_{k,t+1}D_{k,t,t+1}^H)^{-1})] &= 0 \\
E_t[\mathcal{N}_{k,t+1}^{-1}\Psi_{k,t,t+1}^H\mathcal{N}_{k,t+1}D_{k,t,t+1}^H(\Delta W_{k,t,t+1}^{*,I/H} + \lambda_{k,t}^{(1)} + \mu_{k,t}(\mathcal{N}_{k,t+1}D_{k,t,t+1}^H)^{-1})] &= 0,
\end{aligned} \tag{C.12}$$

and solving this system we arrive at the required result.

Q.E.D.

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<sup>74</sup>The dependence on  $t$  arises due to the finite horizon  $T$  and vanishes as  $T \rightarrow \infty$ .

**Proof of Proposition 13.** Recall that

$$\mathcal{N}_{i,t+1}^{-1} = e^{\alpha^{\mathcal{N}} \delta \omega_{t+1} - \varepsilon_{i,t+1}^{\mathcal{N}}}$$

Since all expressions are homogeneous of degree zero in  $E_t[e^{\varepsilon_{i,t+1}^{\mathcal{N}}}]$ , we can impose the normalization  $E_t[\varepsilon_{i,t+1}^{\mathcal{N}}] = 1$ . Under the independence assumption and the identical discount factors assumption, we have that the solution to (C.12) is given by

$$\begin{aligned} \mu_{i,t} &= \frac{E_t[\mathcal{N}_{i,t+1}^{-1} \Psi_{i,t+1}] E_t[\frac{\bar{S}_{t+1}^{\$}}{S_{i,t+1}^{\$}} D_{i,t+1} \Psi_{i,t+1}] - E_t[\mathcal{N}_{i,t+1}^{-1} \Psi_{i,t+1} \frac{\bar{S}_{t+1}^{\$}}{S_{i,t+1}^{\$}}] E_t[D_{i,t+1} \Psi_{i,t+1}]}{E_t[\Psi_{i,t+1} D_{i,t+1}] E_t[\Psi_{i,t+1} (D_{i,t+1})^{-1} \mathcal{N}_{i,t+1}^{-2}] - (E_t[\Psi_{i,t+1} \mathcal{N}_{i,t+1}^{-1}])^2} \\ &= \frac{E_t[\Psi_{i,t+1}] E_t[e^{\alpha^{\mathcal{N}} \delta \omega_{t+1}} \frac{\bar{S}_{t+1}^{\$}}{S_{i,t+1}^{\$}} D_{i,t+1} \Psi_{i,t+1}] - E_t[e^{\alpha^{\mathcal{N}} \delta \omega_{t+1}} \Psi_{i,t+1} \frac{\bar{S}_{t+1}^{\$}}{S_{i,t+1}^{\$}}] E_t[D_{i,t+1} \Psi_{i,t+1}]}{E_t[e^{2\varepsilon_{i,t+1}^{\mathcal{N}}}] E_t[\Psi_{i,t+1} D_{i,t+1}] E_t[\Psi_{i,t+1} (D_{i,t+1})^{-1} e^{2\alpha^{\mathcal{N}} \delta \omega_{t+1}}] - (E_t[\Psi_{i,t+1} e^{\alpha^{\mathcal{N}} \delta \omega_{t+1}}])^2} \\ &= \frac{\alpha}{\text{Var}_t[e^{\varepsilon_{i,t+1}^{\mathcal{N}}}] \beta + \gamma} \end{aligned}$$

for some constants  $\beta, \gamma > 0$  that are independent of the country identity, while the sign of  $\alpha$  depends on whether the policy is mild. At the same time,

$$\begin{aligned} \lambda_{i,t} &\approx 1 + \frac{\bar{S}_{t+1}^{\$}}{S_{i,t+1}^{\$}} - \frac{\widetilde{\text{Cov}}_t(S_{i,t+1}^*, \frac{\bar{S}_{t+1}^{\$}}{S_{i,t+1}^{\$}}, 1/S_{i,t+1}^*)}{\widetilde{\text{Cov}}_t(S_{i,t+1}^*, 1/S_{i,t+1}^*)} \\ &= 1 + \frac{\bar{S}_{t+1}^{\$}}{S_{i,t+1}^{\$}} \\ &\quad + \frac{E_t[\mathcal{N}_{i,t+1}^{-1} \Psi_{i,t+1}] E_t[\frac{\bar{S}_{t+1}^{\$}}{S_{i,t+1}^{\$}} \mathcal{N}_{i,t+1}^{-1} \Psi_{i,t+1}] - E_t[D_{i,t+1} \Psi_{i,t+1} \frac{\bar{S}_{t+1}^{\$}}{S_{i,t+1}^{\$}}] E_t[\Psi_{i,t+1} (D_{i,t+1})^{-1} \mathcal{N}_{i,t+1}^{-2}]}{E_t[\Psi_{i,t+1} D_{i,t+1}] E_t[\Psi_{i,t+1} (D_{i,t+1})^{-1} \mathcal{N}_{i,t+1}^{-2}] - (E_t[\Psi_{i,t+1} \mathcal{N}_{i,t+1}^{-1}])^2} \end{aligned}$$

Q.E.D.

**Proof of Proposition 14.** We have

$$\begin{aligned}
& -e^{-r_{\$,t}^{H,i}} + e^{-r_{\$,t}} = -E_t[M_{i,t,t+1}^H(\mathcal{E}_{i,t}/\mathcal{E}_{i,t+1})] + E_t[M_{\$,t,t+1}^H] \\
& \approx -E_t[M_{i,t,t+1}^{H,*}(1 + M_{i,t,t+1}^{H,(1)}) \frac{M_{\$,t,t+1}^*}{M_{i,t,t+1}^*}(1 + M_{\$,t,t+1}^{I,(1)} - M_{i,t,t+1}^{I,(1)})] + E_t[M_{\$,t,t+1}^{H,*}(1 + M_{\$,t,t+1}^{H,(1)})] \\
& \approx E_t[M_{\$,t,t+1}^{H,*}(M_{\$,t,t+1}^{H,(1)} - M_{i,t,t+1}^{H,(1)} - (M_{\$,t,t+1}^{I,(1)} - M_{i,t,t+1}^{I,(1)}))] \\
& = E_t \left[ M_{\$,t,t+1}^{H,*} \left( M_{\$,t,t+1}^{H,(1)} - M_{i,t,t+1}^{H,(1)} - 2(M_{\$,t,t+1}^{I,(1)} - M_{i,t,t+1}^{I,(1)}) + \theta_{\$} \Delta C_{t,t+1}^{H/\$} - \theta_i \Delta C_{t,t+1}^{H/i} \right) \right] \\
& = E_t \left[ M_{\$,t,t+1}^{H,*} \left( \bar{\theta}_{\$} \Delta \bar{C}_{t,t+1}^{H/\$,*} - \frac{C_{\$,0}^I}{2C_{\$,0}^I + C_{\$,0}^H} (\bar{\theta}_{\$} \Delta \bar{W}_{t,t+1}^{H/\$,*} + \lambda_{\$,t}^{(1)} + \mu_{\$,t}(\mathcal{N}_{\$,t+1} D_{\$,t,t+1}^H)^{-1}) \right. \right. \\
& \quad - \bar{\theta}_i \Delta \bar{C}_{t,t+1}^{H/i,*} + \frac{C_{i,0}^I}{2C_{i,0}^I + C_{i,0}^H} (\bar{\theta}_i \Delta \bar{W}_{t,t+1}^{H/i,*} + \lambda_{i,t}^{(1)} + \mu_{i,t}(\mathcal{N}_{i,t+1} D_{i,t,t+1}^H)^{-1}) \\
& \quad - 2 \left( \bar{\theta}_{\$} \Delta \bar{C}_{t,t+1}^{H/\$,*} + \frac{C_{\$,0}^H}{2C_{\$,0}^I + C_{\$,0}^H} (\bar{\theta}_{\$} \Delta \bar{W}_{t,t+1}^{H/\$,*} + \lambda_{\$,t}^{(1)} + \mu_{\$,t}(\mathcal{N}_{\$,t+1} D_{\$,t,t+1}^H)^{-1}) \right. \\
& \quad \left. \left. - \bar{\theta}_i \Delta \bar{C}_{t,t+1}^{H/i,*} - \frac{C_{i,0}^H}{2C_{i,0}^I + C_{i,0}^H} (\bar{\theta}_i \Delta \bar{W}_{t,t+1}^{H/i,*} + \lambda_{i,t}^{(1)} + \mu_{i,t}(\mathcal{N}_{i,t+1} D_{i,t,t+1}^H)^{-1}) \right) \right. \\
& \quad \left. + \theta_{\$} \Delta C_{t,t+1}^{H/\$} - \theta_i \Delta C_{t,t+1}^{H/i} \right) \right] \\
& = E_t \left[ \frac{C_{i,0}^I + 2C_{i,0}^H}{2C_{i,0}^I + C_{i,0}^H} (\bar{\theta}_i \Delta \bar{W}_{t,t+1}^{H/i,*} + \lambda_{i,t}^{(1)} + \mu_{i,t}(\mathcal{N}_{i,t+1} D_{i,t,t+1}^H)^{-1}) \right. \\
& \quad \left. - \frac{C_{\$,0}^I + 2C_{\$,0}^H}{2C_{\$,0}^I + C_{\$,0}^H} (\bar{\theta}_{\$} \Delta \bar{W}_{t,t+1}^{H/\$,*} + \lambda_{\$,t}^{(1)} + \mu_{\$,t}(\mathcal{N}_{\$,t+1} D_{\$,t,t+1}^H)^{-1}) \right]
\end{aligned}$$

Suppose first that there is no noise in monetary policy. Using the approximation

$$E[X] = E[e^{\log X}] \approx e^{E[\log X] + 0.5 \text{Var}[\log X]} \approx e^{E[\log X]} (1 + 0.5 \text{Var}[\log X])$$

that holds in the limit of small variance, we get

$$\frac{\bar{S}_{t+1}^{\$}}{S_{i,t+1}^{\$}} \approx e^{-\delta_i^S \omega_{t+1}} = e^{-\delta_i^S / \delta_i \log D_{i,t+1}}$$

and hence, defining  $\alpha_i^I = -\delta_i^S/\delta_i$ , we get

$$\begin{aligned}
\mu_{i,t} &= \frac{E_t[\mathcal{N}_{i,t+1}^{-1}\Psi_{i,t+1}]E_t[\frac{\bar{S}_{t+1}^\$}{S_{i,t+1}^\$}D_{i,t,t+1}\Psi_{i,t+1}] - E_t[\mathcal{N}_{i,t+1}^{-1}\Psi_{i,t+1}\frac{\bar{S}_{t+1}^\$}{S_{i,t+1}^\$}]E_t[D_{i,t,t+1}\Psi_{i,t+1}]}{E_t[\Psi_{i,t+1}D_{i,t,t+1}]E_t[\Psi_{i,t+1}(D_{i,t,t+1})^{-1}\mathcal{N}_{i,t+1}^{-2}] - (E[\Psi_{i,t+1}\mathcal{N}_{t+1}^{-1}])^2} \\
&= \frac{\bar{S}_t^\$}{S_{i,t}^\$} \frac{E_t[e^{\psi+\alpha_i^\mathcal{N}d}]E_t[e^{d+\alpha_i^Id+\psi}] - E_t[e^{\alpha_i^\mathcal{N}d+\psi+\alpha_i^Id}]E_t[e^{d+\psi}]}{E_t[e^{\psi+d}]E_t[e^{\psi-d+2\alpha_i^\mathcal{N}d}] - E_t[e^{\psi+\alpha_i^\mathcal{N}d}]^2} \\
&= \frac{\bar{S}_t^\$}{S_{i,t}^\$} \frac{\text{Var}_t[\psi + \alpha_i^\mathcal{N}d] + \text{Var}_t[(1 + \alpha_i^I)d + \psi] - \text{Var}_t[(\alpha_i^\mathcal{N} + \alpha_i^I)d + \psi] - \text{Var}_t[d + \psi]}{\text{Var}_t[\psi + d] + \text{Var}_t[\psi + (2\alpha_i^\mathcal{N} - 1)d] - 2\text{Var}_t[\psi + \alpha_i^\mathcal{N}d]} \\
&= \frac{\bar{S}_t^\$}{S_{i,t}^\$} \frac{(\alpha_i^\mathcal{N})^2 + (1 + \alpha_i^I)^2 - (\alpha_i^\mathcal{N} + \alpha_i^I)^2 - 1}{1 + (2\alpha_i^\mathcal{N} - 1)^2 - 2(\alpha_i^\mathcal{N})^2} = \frac{\bar{S}_t^\$}{S_{i,t}^\$} \frac{\alpha_i^I}{1 - \alpha_i^\mathcal{N}}
\end{aligned}$$

Similarly,

$$\begin{aligned}
&\frac{\widetilde{\text{Cov}}_t(S_{i,t+1}^* \frac{\bar{S}_{t+1}^\$}{S_{i,t+1}^\$}, 1/S_{i,t+1}^*)}{\widetilde{\text{Cov}}_t(S_{i,t+1}^*, 1/S_{i,t+1}^*)} \\
&= \frac{E_t[\mathcal{N}_{i,t+1}^{-1}\Psi_{i,t+1}]E_t[\frac{\bar{S}_{t+1}^\$}{S_{i,t+1}^\$}\mathcal{N}_{i,t+1}^{-1}\Psi_{i,t+1}] - E_t[D_{i,t+1}\Psi_{i,t+1}\frac{\bar{S}_{t+1}^\$}{S_{i,t+1}^\$}]E_t[\Psi_{i,t+1}(D_{i,t+1})^{-1}\mathcal{N}_{i,t+1}^{-2}]}{E_t[\Psi_{i,t+1}D_{i,t+1}]E_t[\Psi_{i,t+1}(D_{i,t+1})^{-1}\mathcal{N}_{i,t+1}^{-2}] - (E[\Psi_{i,t+1}\mathcal{N}_{t+1}^{-1}])^2} \\
&\approx \frac{\bar{S}_t^\$}{S_{i,t}^\$} \frac{\text{Var}_t[\psi + \alpha_i^\mathcal{N}d] + \text{Var}_t[(\alpha_i^\mathcal{N} + \alpha_i^I)d + \psi] - \text{Var}_t[(1 + \alpha_i^I)d + \psi] - \text{Var}_t[(2\alpha_i^\mathcal{N} - 1)d + \psi]}{\text{Var}_t[\psi + d] + \text{Var}_t[\psi + (2\alpha_i^\mathcal{N} - 1)d] - 2\text{Var}_t[\psi + \alpha_i^\mathcal{N}d]} \\
&= \frac{\bar{S}_t^\$}{S_{i,t}^\$} \frac{(\alpha_i^\mathcal{N})^2 + (\alpha_i^\mathcal{N} + \alpha_i^I)^2 - (1 + \alpha_i^I)^2 - (2\alpha_i^\mathcal{N} - 1)^2}{2(\alpha_i^\mathcal{N} - 1)^2} = \frac{\bar{S}_t^\$}{S_{i,t}^\$} \frac{\alpha_i^I - \alpha_i^\mathcal{N} + 1}{\alpha_i^\mathcal{N} - 1},
\end{aligned}$$

and hence

$$\begin{aligned}
\lambda_{i,t} &\approx 1 + \theta_i \left( \frac{\bar{S}_t^\$}{S_{i,t}^\$} - \frac{\widetilde{\text{Cov}}_t\left(S_{i,t+1}^* \frac{\bar{S}_{t+1}^\$}{S_{i,t+1}^\$}, 1/S_{i,t+1}^*\right)}{\widetilde{\text{Cov}}_t(S_{i,t+1}^*, 1/S_{i,t+1}^*)} \right) \\
&= 1 + \theta_i \frac{\bar{S}_t^\$}{S_{i,t}^\$} \left( 1 + \frac{\alpha_i^I - \alpha_i^\mathcal{N} + 1}{\alpha_i^\mathcal{N} - 1} \right) = 1 + \theta_i \frac{\bar{S}_t^\$}{S_{i,t}^\$} \frac{\alpha_i^I}{\alpha_i^\mathcal{N} - 1}.
\end{aligned}$$

If  $C_0^I/C_0^H$  is the same across the two countries, then

$$\begin{aligned}
& -e^{-r_{\$,t}^{H,i}} + e^{-r_{\$,t}} \\
& \approx \frac{C_0^I + 2C_0^H}{2C_0^I + C_0^H} \bar{\theta} E_t \left[ \mathcal{N}_{\$,t+1}^{-1} \Psi_{\$,t,t+1} \left( (\Delta W_{t,t+1}^{*,H/i} - \Delta W_{t,t+1}^{*,H/\$}) \right. \right. \\
& \left. \left. + \left( \frac{\bar{S}_t^\$}{S_{i,t}^\$} \frac{\alpha_i^I}{\alpha_i^\mathcal{N} - 1} (1 - (D_{i,t,t+1})^{\alpha_i^\mathcal{N} - 1}) - \frac{\bar{S}_t^\$}{S_{\$,t}^\$} \frac{\alpha_\$^I}{\alpha_\$^\mathcal{N} - 1} (1 - (D_{\$,t,t+1})^{\alpha_\$^\mathcal{N} - 1}) \right) \right] \right].
\end{aligned}$$

If  $D_{i,t,t+1} = D_{\$,t,t+1}$ , then we get

$$\begin{aligned}
& -e^{-r_{\$,t}^{H,i}} + e^{-r_{\$,t}} \\
& \approx \frac{C_0^I + 2C_0^H}{2C_0^I + C_0^H} \bar{\theta} E_t \left[ \mathcal{N}_{\$,t+1}^{-1} \Psi_{\$,t,t+1} \left( W_t^{*,H/i} \alpha^I (F(\alpha_i^\mathcal{N}, D_{t,t+1}) - F(\alpha_\$^\mathcal{N}, D_{t,t+1})) \right) \right].
\end{aligned}$$

with  $F(\alpha, x) = (1 - x^{\alpha-1})/(\alpha - 1)$  and the claim follows because  $F$  is monotone decreasing in  $\alpha$  for  $x$  close to one.<sup>75</sup> More generally, substituting

$$D_{i,t,t+1} = e^{\delta_i \Delta \omega_{t+1}}, \quad \left( \frac{S_{i,t+1}^\$}{\bar{S}_{t+1}^\$} \right) \frac{\bar{S}_t^\$}{S_{i,t}^\$} = e^{\delta_i^S \Delta \omega_{t+1}},$$

and denoting

$$Q_i = \bar{\theta}_i \frac{C_{i,0}^I + 2C_{i,0}^H}{2C_{i,0}^I + C_{i,0}^H},$$

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<sup>75</sup>Since  $D_{i,t,t+1}$  has a low variance, it is close to one with a probability close to one.

we get

$$\begin{aligned}
& -e^{-r_{\$,t}^{H,i}} + e^{-r_{\$,t}} \\
& \approx E_t \left[ \mathcal{N}_{\$,t+1}^{-1} \Psi_{\$,t,t+1} \left( (Q_i \Delta W_{t,t+1}^{*,H/i} - Q_{\$} \Delta W_{t,t+1}^{*,H/\$}) \right. \right. \\
& + \left. \left. \left( \frac{\bar{S}_t^{\$}}{S_{i,t}^{\$}} Q_i \frac{\alpha_i^I}{\alpha_i^{\mathcal{N}} - 1} (1 - (D_{i,t,t+1})^{\alpha_i^{\mathcal{N}} - 1}) - W_t^{*,H/\$} Q_{\$} \frac{\alpha_{\$}^I}{\alpha_{\$}^{\mathcal{N}} - 1} (1 - (D_{\$,t,t+1})^{\alpha_{\$}^{\mathcal{N}} - 1}) \right) \right) \right] \\
& = \frac{\bar{S}_t^{\$}}{S_{\$,t}^{\$}} E_t \left[ \mathcal{N}_{\$,t+1}^{-1} \Psi_{\$,t,t+1} \left( Q_i \frac{S_{\$,t}^{\$}}{S_{i,t}^{\$}} (e^{-\delta_i^S \Delta \omega_{t+1}} - 1) - Q_{\$} (e^{-\delta_{\$}^S \Delta \omega_{t+1}} - 1) \right) \right. \\
& + \left. \left( \frac{S_{\$,t}^{\$}}{S_{i,t}^{\$}} Q_i \frac{\alpha_i^I}{\alpha_i^{\mathcal{N}} - 1} (1 - e^{\delta_i \Delta \omega_{t+1} (\alpha_i^{\mathcal{N}} - 1)}) - Q_{\$} \frac{\alpha_{\$}^I}{\alpha_{\$}^{\mathcal{N}} - 1} (1 - e^{\delta_{\$} \Delta \omega_{t+1} (\alpha_{\$}^{\mathcal{N}} - 1)}) \right) \right].
\end{aligned}$$

And we can rewrite it as

$$\begin{aligned}
& -e^{-r_{\$,t}^{H,i}} + e^{-r_{\$,t}} \\
& \approx E_t \left[ \mathcal{N}_{\$,t+1}^{-1} \Psi_{\$,t,t+1} \left( (Q_i \Delta W_{t,t+1}^{*,H/i} - Q_{\$} \Delta W_{t,t+1}^{*,H/\$}) \right. \right. \\
& + \left. \left. \left( \frac{\bar{S}_t^{\$}}{S_{i,t}^{\$}} Q_i \frac{\alpha_i^I}{\alpha_i^{\mathcal{N}} - 1} (1 - (D_{i,t,t+1})^{\alpha_i^{\mathcal{N}} - 1}) - W_t^{*,H/\$} Q_{\$} \frac{\alpha_{\$}^I}{\alpha_{\$}^{\mathcal{N}} - 1} (1 - (D_{\$,t,t+1})^{\alpha_{\$}^{\mathcal{N}} - 1}) \right) \right) \right] \\
& = \frac{\bar{S}_t^{\$}}{S_{\$,t}^{\$}} E_t \left[ \mathcal{N}_{\$,t+1}^{-1} \Psi_{\$,t,t+1} \left( Q_i \frac{S_{\$,t}^{\$}}{S_{i,t}^{\$}} (e^{-\delta_i^S \Delta \omega_{t+1}} - 1) - Q_{\$} (e^{-\delta_{\$}^S \Delta \omega_{t+1}} - 1) \right) \right. \\
& + \left. \left( \frac{S_{\$,t}^{\$}}{S_{i,t}^{\$}} Q_i \frac{\alpha_i^I}{\alpha_i^{\mathcal{N}} - 1} (1 - e^{\delta_i \Delta \omega_{t+1} (\alpha_i^{\mathcal{N}} - 1)}) - Q_{\$} \frac{\alpha_{\$}^I}{\alpha_{\$}^{\mathcal{N}} - 1} (1 - e^{\delta_{\$} \Delta \omega_{t+1} (\alpha_{\$}^{\mathcal{N}} - 1)}) \right) \right] \\
& = \frac{\bar{S}_t^{\$}}{S_{\$,t}^{\$}} E_t \left[ \mathcal{N}_{\$,t+1}^{-1} \Psi_{\$,t,t+1} \left( Q_i \frac{S_{\$,t}^{\$}}{S_{i,t}^{\$}} \left( (e^{-\delta_i^S \Delta \omega_{t+1}} - 1) + \frac{-\delta_i^S / \delta_i}{\alpha_i^{\mathcal{N}} - 1} (1 - e^{\delta_i \Delta \omega_{t+1} (\alpha_i^{\mathcal{N}} - 1)}) \right) \right. \right. \\
& - \left. \left. Q_{\$} \left( (e^{-\delta_{\$}^S \Delta \omega_{t+1}} - 1) + \frac{-\delta_{\$}^S / \delta_{\$}}{\alpha_{\$}^{\mathcal{N}} - 1} (1 - e^{\delta_{\$} \Delta \omega_{t+1} (\alpha_{\$}^{\mathcal{N}} - 1)}) \right) \right) \right] \\
& = \frac{\bar{S}_t^{\$}}{S_{\$,t}^{\$}} E_t \left[ \mathcal{N}_{\$,t+1}^{-1} \Psi_{\$,t,t+1} \left( Q_i \frac{S_{\$,t}^{\$}}{S_{i,t}^{\$}} \left( (e^{-\delta_i^S \Delta \omega_{t+1}} - 1) + \frac{-\delta_i^S / \delta_i}{\alpha_i^{\mathcal{N}} - 1} (1 - e^{\delta_i \Delta \omega_{t+1} (\alpha_i^{\mathcal{N}} - 1)}) \right) \right. \right. \\
& - \left. \left. Q_{\$} \left( (e^{-\delta_{\$}^S \Delta \omega_{t+1}} - 1) + \frac{-\delta_{\$}^S / \delta_{\$}}{\alpha_{\$}^{\mathcal{N}} - 1} (1 - e^{\delta_{\$} \Delta \omega_{t+1} (\alpha_{\$}^{\mathcal{N}} - 1)}) \right) \right) \right] \\
& = \frac{\bar{S}_t^{\$}}{S_{\$,t}^{\$}} E_t \left[ \mathcal{N}_{\$,t+1}^{-1} \Psi_{\$,t,t+1} \left( \delta_i^S Q_i \frac{S_{\$,t}^{\$}}{S_{i,t}^{\$}} \left( -F(-\delta_i^S) + F(\delta_i (\alpha_i^{\mathcal{N}} - 1)) \right) \right. \right. \\
& - \left. \left. \delta_{\$}^S Q_{\$} \left( -F(-\delta_{\$}^S) + F(\delta_{\$} (\alpha_{\$}^{\mathcal{N}} - 1)) \right) \right) \right] \\
& \approx 0.5 \frac{\bar{S}_t^{\$}}{S_{\$,t}^{\$}} \left( \delta_i^S Q_i \frac{S_{\$,t}^{\$}}{S_{i,t}^{\$}} (\delta_i^S + \delta_i (\alpha_i^{\mathcal{N}} - 1)) - \delta_{\$}^S Q_{\$} (\delta_{\$} (\alpha_{\$}^{\mathcal{N}} - 1) + \delta_{\$}^S) \right) E_t [(\Delta \omega_{t+1})^2],
\end{aligned}$$

where we have used the Taylor approximation

$$F(\alpha) = (e^{\Delta \omega_{t+1} \alpha} - 1) / \alpha \approx \frac{\Delta \omega_{t+1} \alpha + 0.5(\Delta \omega_{t+1} \alpha)^2}{\alpha} = \Delta \omega_{t+1} + 0.5 \alpha (\Delta \omega_{t+1})^2.$$

Q.E.D.

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